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Mathematical Modeling of Microwave Heating in a Slab: Taylor Series Method

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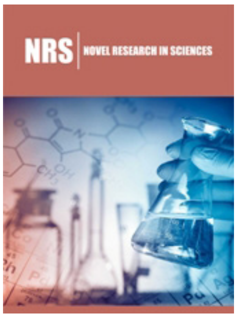

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Abstract

Mathematical modelling of microwave heating in a slab is discussed. In simulating microwave heating in an infinite slab with isothermal walls, a steady-state, non-linear reaction-diffusion equation with source terms occurs. We explicitly develop the approximate solution of this equation using the Taylor series method. The approximate analytical solution is compared with simulation results (Matlab programme) for all experimental values of parameters. A satisfactory agreement is noted. The effect of the parameters thermal absorptivity, electric field decay rate and thermal absorptivity index on temperature field is discussed.

Keywords: Mathematical modelling; Nonlinear equation; Infinite slab; Microwave heating model; Numerical simulation; Taylor series method

Introduction

Microwaves are a type of electromagnetic radiation, which are waves of electrical and magnetic energy travelling through space. The spectrum of electromagnetic radiation includes both powerful X-rays and weaker radio frequency waves that are employed in broadcasting. The radio frequency band of electromagnetic radiation includes microwaves.

Microwave heating has found widespread use in the energy, construction, forestry, chemical and food industries, etc. Three features of microwaves make it possible to employ them for cooking: they are absorbed by metal and passthrough materials like glass, paper and plastic [1]. Numerous industrial processes, including those involving melting, smelting, sintering, drying, and joining, have found new uses for this technology [2].

Heating by microwave radiation is a reaction-diffusion problem with a radiative heat source term, and due to the long-term behaviour of the solutions in space, the system may develop hotspots or localised areas of excessive heating [3]. To solve this problem, several numerical techniques, including finite differences, the spectrum method, the gunshot method, etc., have emerged in recent decades. With the introduction of unique hybrid numerical-analytical systems for nonlinear differential equations, the concepts of classical analytical methods have recently undergone a renaissance.

To analyse the mathematical model to get insight into an essentially complicated physical process to forecast the occurrence of such events. The theory of response diffusion equations is exceedingly complex, and it is still very difficult to solve these equations in rectangular, cylindrical, and spherical coordinates with any practical relevance in the engineering sciences. With the development of new hybrid numerical-analytical systems for nonlinear differential equations, the concepts of classical analytical methods have recently done revival.

Hermite-approximation Pade's approach is one such trend [4-6]. Makinde [7] applies Hermite-approximation Pade's method for solving the steady-state nonlinear equation in a temperature field. For a better understanding of these processes and the development of highly efficient microwave installations, simple mathematical modeling is needed. In this paper, we obtain the simple analytical expression for the temperature field by solving the nonlinear equation using the Taylor series method.

Methods and Material

The steady-state nonlinear diffusion equation with the source term, which describes the thermal behaviour of the basic microwave heating model can be expressed as follows [8]:

$$\alpha \frac{d^2T(y)}{dy^2} + Ee^{-\beta y} T(y)^n = 0 \quad (1)$$

The boundary conditions are

At $y = 0, \frac{dT}{dy} = 0$ (2)

At $y = a, T = T_0$ (3)

where T is the temperature, γ is the distances measured in the normal direction, E represents the amplitude of the incident radiation, α denotes the thermal constant, β is the electric field amplitude decay rate and n is the thermal absorptivity index, a is the slab of half width and T_0 is the wall temperature. By introducing the following dimensionless variables

$$Y = \frac{y}{a}, u = \frac{T}{T_0}, \lambda = \frac{Ea^2 T_0^{n-1}}{\alpha}, k = \alpha\beta \quad (4)$$

the nonlinear differential equation (1) becomes the dimensionless form as follows:

$$\frac{d^2u(Y)}{dY^2} + \lambda e^{-kY} u(Y)^n = 0 \quad (5)$$

The corresponding dimensionless boundary conditions are

At $Y = 0, \frac{du}{dy} = 0$ (6)

At $Y = 1, u = 1$ (7)

whereis the dimensionless temperature field, λ is the thermal absorptivity parameter, k is the electric filed decay rateand Y is the dimensionless distance.

Results and Discussion

Numerical and analytical solutions of the ordinary/partial differential equations (ODEs) by using the Taylor series method have been investigated by many authors [9-21] and references therein. The dimensionless temperature field can be obtained by solving the equation (5) using Taylors's series method as follows:

$$u(Y) = u(0) - \lambda(u(0))^n \frac{Y^2}{2!} + \lambda k(u(0))^n \frac{Y^3}{3!} - \left(\lambda k^2(u(0))^n + \lambda^2 n(u(0))^{2n-1}\right)\frac{Y^4}{4!} \quad (8)$$

Using the boundary condition, $u(1)=1$ in Eq. (9) gives the numerical value of $u(0)$. For examples the experimental values of the parameters $\lambda = 0.1, k = 0.1$,and $n = 0.1$ and $u(0) = 1.04865$. The analytical expression for the temperature field takes on the form

$$u(Y) = 1.04865 - 0.02021Y^2 + 0.00168Y^3 - 0.00008Y^4 \quad (9)$$

Hermite- Pade´ approximation method was applied by Makinde [7] to solve Eq. (5) with boundary conditions (6) and (7). The analytical expression for the temperature field is obtained

$$u(Y) = 1 - \frac{\lambda}{k_2}(kY - k + e^{-kY} - e^{-k}) - \frac{\lambda^2 ne^{-k}}{4k^4}\begin{pmatrix} 4kY + 4k^2Ye^k - 6kYe^k - 4k - 3e^{-k} + 6ke^{-k} \\ -4k^2e^k + 8 + ke^{-k(Y-1)} + 4e^{-kY} - \\ 8e^{-k(Y-1)} - 4kYe^{-k(Y-1)} - e^{-k(2Y-1)} \end{pmatrix} \quad (10)$$

Equations (6) and (7) numerically solve the nonlinear Eq. (5) in the temperature field for the boundary condition. The Scala/Matlab (Appendix B) software's function pdex1 was used to numerically solve problems including initial-boundary values for non-linear differential equations. The present and earlier analytical results are compared to this numerical solution in Tables (1-3). A satisfactory agreement is indicated.

Table 1: Comparison of dimensionless temperature field u(Y) with simulation results for various values of parameter λ when "k=0.1" and "n=0.1".

y	$\lambda = 0.1, u(0) = 1.04865$			$\lambda = 0.5, u(0) = 1.24817$			$\lambda = 1, u(0) = 1.507$		
	Num.	Analytical Eq. (8)	Error %	Num	Analytical Eq. (8)	Error %	Num	Analytical Eq. (8)	Error %
0	1.049	1.049	0	1.246	1.248	0.16	1.501	1.507	0.4
0.2	1.047	1.047	0	1.236	1.348	0.162	1.48	1.486	0.405
0.4	1.04	1.041	0.096	1.205	1.207	0.166	1.417	1.423	0.423
0.6	1.03	1.031	0.097	1.155	1.156	0.087	1.314	1.319	0.38
0.8	1.017	1.017	0	1.084	1.085	0.092	1.171	1.175	0.341
1	1	1	0	1	1	0	1	1	0
	Average error (%)		0.032	Average error (%)		0.111	Average error (%)		0.325

Table 2: Comparison of dimensionless temperature field u(Y) with simulation results for various values of parameterk when λ”=0.1” and “n=0.1”.

y	$k=0.5, u(0)=1.04293$			$k=1, u(0)=1.03768$			$k=1.5, u(0)=1.03453$		
	Num.	Analytical Eq. (8)	Error %	Num	Analytical Eq. (8)	Error %	Num	Analytical Eq. (8)	Error %
0	1.043	1.043	0	1.037	1.038	0.096	1.032	1.035	0.291
0.2	1.041	1.041	0	1.035	1.036	0.097	1.03	1.033	0.291
0.4	1.035	1.035	0	1.03	1.03	0	1.025	1.028	0.293
0.6	1.026	1.026	0	1.022	1.022	0	1.018	1.02	0.196
0.8	1.014	1.014	0	1.011	1.012	0.099	1.01	1.011	0.099
1	1	1	0	1	1	0	1	1	0
	Average error (%)		0	Average error (%)		0.049	Average error (%)		0.195

Table 3: Comparison of dimensionless temperature field u(Y) with simulation results and previous analytical results for k=1,and n=0.1.

y	$\lambda=0.1, u(0)=1.03768$					$\lambda=0.3, u(0)=1.11407$				
	Num.	TSM Eq. (8). This work	Makinde [7] Eq. (10)	Error % TSM Eq. (8). This work	Error % Makinde [7] Eq. (10)	Num.	TSM Eq. (8). This work	Makinde [7] Eq. (10)	Error % TSM Eq. (8). This work	Error % Makinde [7] Eq. (10)
0	1.037	1.038	1.039	0.096	0.193	1.111	1.114	1.13	0.27	1.71
0.2	1.035	1.036	1.037	0.097	0.193	1.106	1.108	1.123	0.181	1.537
0.4	1.03	1.03	1.032	0	0.194	1.09	1.092	1.106	0.183	1.468
0.6	1.022	1.022	1.023	0	0.098	1.065	1.068	1.081	0.282	1.502
0.8	1.011	1.012	1.013	0.099	0.198	1.035	1.036	1.049	0.097	1.353
1	1	1	1	0	0	1	1	1	0	0
	Average error (%)			0.048	0.146	Average error (%)			0.169	1.263

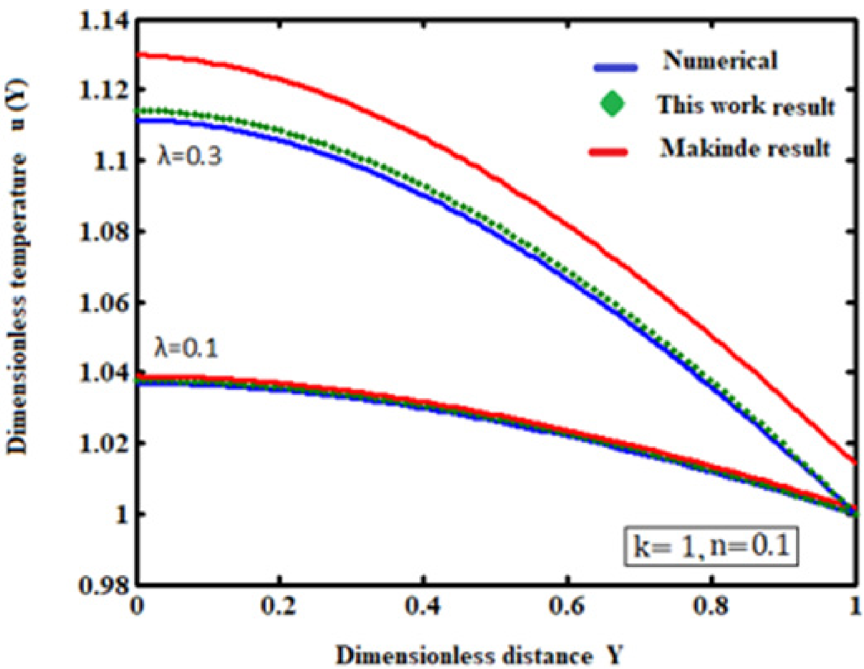

Figure 1: Comparison of analytical expression of temperature field $u(Y)$ with simulation results for various value of parameter λ using eq. (8) and Makinde result using the eq. (10).

Equation (8) represents the approximate analytical expression of the temperature profile. The parameters λ, k and n which are fundamental for applications in industrial safety and explosives handling techniques, determine the thermal decomposition of the reacting combustible material. Our analytical results are compared with simulations results and previous analytical results for various values of λ in Figure 1. Figure 2(a-c) represents the effects of the thermal absorptivity, thermal absorptivity index, and Electric field decay rate parameter on a temperature profile for the various reaction mechanism. From the figure, it is observed that the temperature increases with increasing the thermal absorptivity. From the figure, it is observed that temperature decreases, and the electric field decreases.

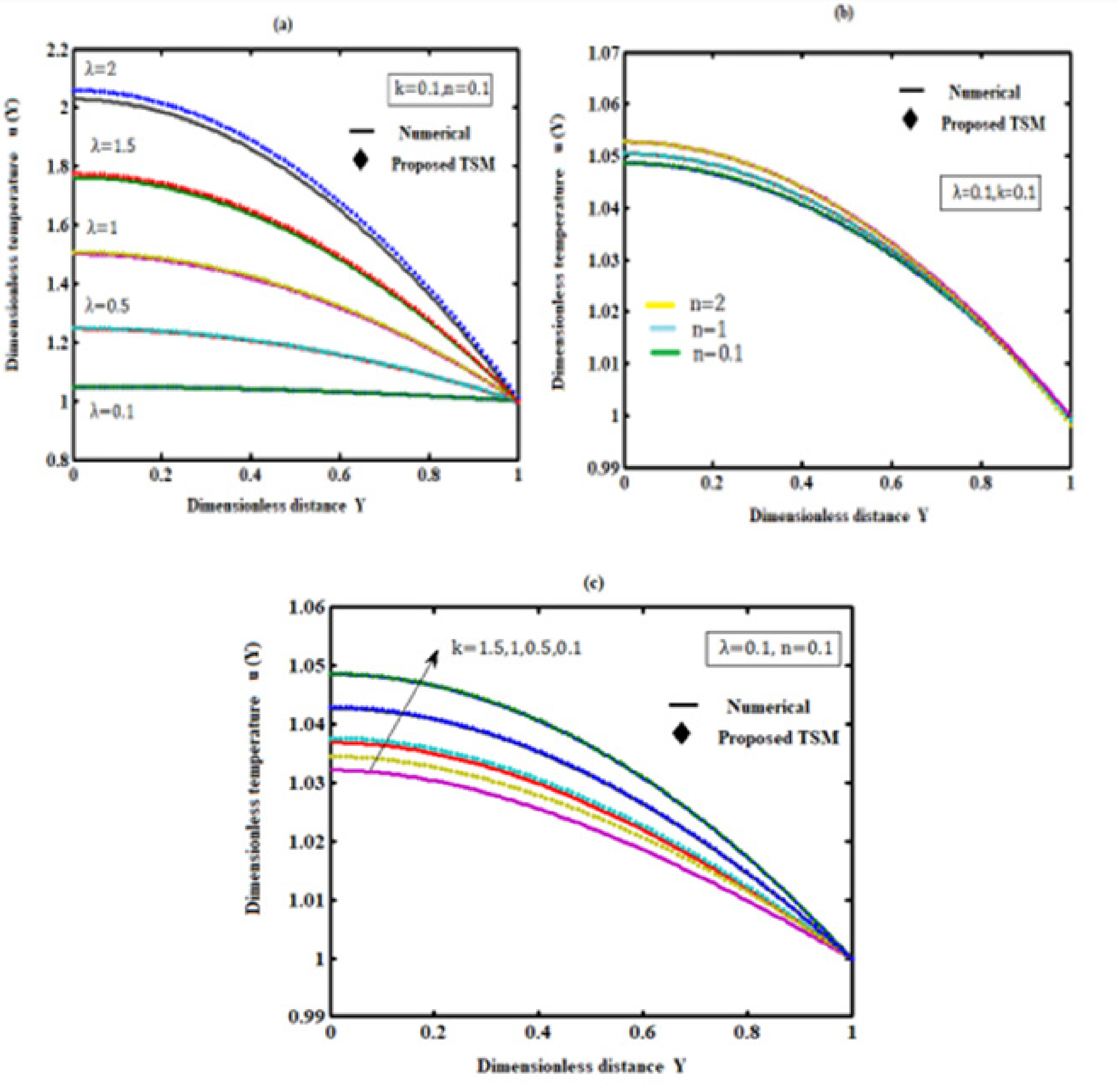

Figure 2: Comparison of analytical expression of temperature field $u(Y)$ with numerical results for various value of parameters λ, k and n using eq. (8).

Conclusion

We studied the microwave heading model in the slab under thermal absorptivity, thermal absorptivity index, and electric filed decay rate. The steady-state nonlinear diffusion equation in microwave heating in the slab is solved using the analytical method. The simple and new closed form of analytical expression of temperature field is reported. The results illustrate that this method gives a highly accurate and good approximation of the solution to this nonlinear system.

List of symbols

Symbols	Meanings
α	slab of half width
E	Amplitude of the incident radiation
k	Electric filed decay rate parameter
n	Thermal absorptivity index
T	Temperature
T_0	Wall temperature
u	Dimensionless temperature field
y	Distances measured in the normal direction
Y	Dimensionless distance
λ	Thermal absorptivity parameter
α	Thermal constant
β	Electric field amplitude decay rate

Appendix A: Analytical expressions of the temperature field using Taylor series method

Using the Taylor's series, the dimensionless temperature can be expressed as follows:

$$\sum_{i=0}^{m} \frac{d^{iu}}{dY^i}\bigg|_{y=0} \frac{Y^i}{i!} = u(0) + \frac{du}{dY}\bigg|_{Y=0} Y + \frac{d^2u}{dY^2}\bigg|_{Y=0} \frac{Y^2}{2} + \frac{d^3u}{dY}\bigg|_{Y=0} \frac{Y^3}{3!} + \ldots. \quad (A.1)$$

Eq. (5) in the text can be expressed in the form

$$u''(Y) = -\lambda e^{-kY} u(Y)^n \quad (A.2)$$

Taking the derivative of Eq. (A.2)with respect to gives

$$u'''(Y) = -\left(-\lambda k e^{-kY} u(Y)^n + \lambda n e^{-kY} u(Y)^{n-1} u'(Y)\right) \quad (A.3)$$

Substitute in Eq. (A.2) and Eq. (A.3) implies

$$\left(\frac{d^2u}{dY^2}\right)_{Y=0} = -\lambda u(0)^n \quad (A.4)$$

$$\left(\frac{d^{3u}}{dY^3}\right) = \lambda k \left(u(0)\right)^n \quad (A.5)$$

Successive differentiation of Eq. (A.3) gives

$$\left(\frac{d^4u}{dY4}\right)_{Y=0} = -\left(\lambda k^2 (u(0))^n + \lambda^2 n (u(0))^{2n-1}\right) \quad (A.6)$$

Appendix B

Matlab program: Scilab/ Matlab program for the numerical solution of the system of nonlinear Eq.(5)

```
function pdex4
m = 0
x = linspace(0,1)
t= linspace(0,10000)
sol = pdepe(m, @pdex4pde, @pdex4ic, @pdex4bc, x, t)
u1 = sol( :, :,1)
figure
plot(x,u1(end,:))
title('u1(x,t)')
xlabel('Distance x')
ylabel('u1(x,2)')
%-----------------------------------------------------------------
 function [c, f, s] = pdex4pde(x, t, u, DuDx)
c = 1
f = DuDx
a=0.1; b=0.5
F =a*exp(-b*x)*u^(0.1)
s = F
function u0 = pdex4ic(x)
 u0 = 1
function[pl, ql, pr, qr]=pdex4bc(xl, ul, xr, ur, t)
%create a boundary conditions
pl = 0
ql = 1
pr = ur-1
qr = 0
```

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