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Mini Review

Mechanical Fatigue Yield and Lifetime in Engineering

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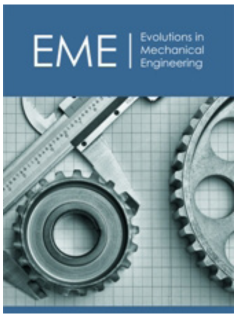

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Introduction

This review presents the link between the fatigue crack growth measurements and the fatigue lifetime prediction. Fracture mechanics focuses on stress fields in materials under cyclic loads at the crack tip. The review tackles finite fatigue yield and lifetime prediction as dynamical finite causal processes that encompass finite cause and effect interactions. The review presents the links between direct fatigue crack growth and the fatigue endurance data in fracture mechanics. In conclusion, fatigue crack growth data are related to the parameters of lifetime S-N curves.

Mechanical Load Redistribution Model of FCG

This review at the beginning briefly recapitulates the elements of Dynamics of Finite Causal Processes (DFCP) and Finite-Cause-and-Effect Interaction (FCEI) [1-7] between the Fatigue Crack Growth (FCG) and the redistribution of cyclic loads among N_f intact micro-structural bonds of material particles. The initial cyclic stress $\sigma_t = F / N_f$ induced by uniaxial constant tensile cyclic force *F* activates in each successive cycle *N* bonds failures. The load redistributes to the remaining (N_f -*N*) and causes overload of $\sigma_N = F / (N_f - N)$. The overstressing rate represents a FCEI process, as:

$$\frac{\sigma_N}{\sigma_t} = \frac{N_f}{N_f - N} = \frac{1}{1-n} \qquad (2.1)$$

The cyclic ratio $n = N/N_f$ is the dimensionless progression of the FCG.

FCEI Model of FCG

FCG is here a FCEI relation between number *N* of cyclic loads of stress range $\Delta\sigma$ (the cause *C*) and increasing crack size a(*N*) (the effect *E*(*C*). The primary FCG starts to grow in proportion p to the cyclic ratio n, as:

$$a_p(n) = p \cdot n \qquad (3.1)$$

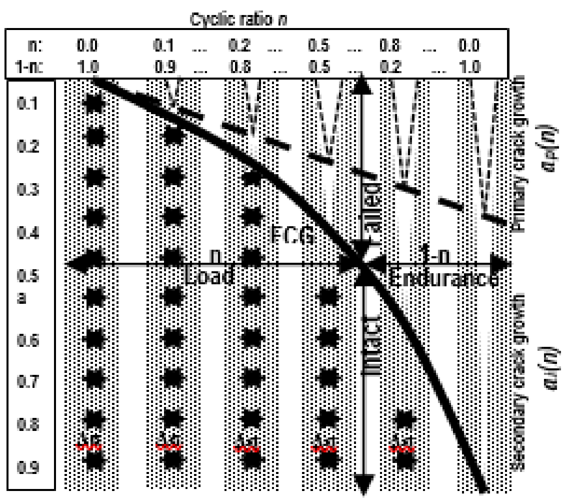

Figure 1: Load redistribution model of FCG.

At the same time, the residual fatigue endurance declines due to load redistribution after *n* load cycles in some proportion *r* to the remaining number of load cycles (1-*n*) (Figure 1) as:

$$e_r(1-n) = r\cdot(1-n) \qquad (3.2)$$

According to FCEI, the rate of change of FCG $a_i(n)$ of interaction of primary FCG (3.1) and drop of endurance (3.2) (Figure 2) due to overstressing and endurance decline in *N* cycles can be mathematically presented as follows:

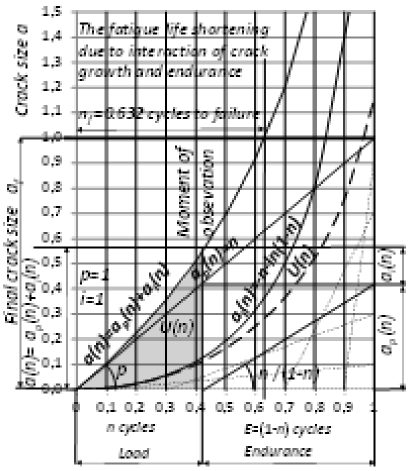

Figure 2: FCEI between cyclic loads n and FCG a(*n*).

$$\frac{\mathrm{d}a_i(n)}{\mathrm{d}n} = \frac{p\cdot n}{r\cdot(1-n)} = i\cdot\frac{n}{1-n} \qquad (3.3)$$

The overall increase of FCG *a*(*n*) consists of the initial growth a_p (3.1) and of the integral of interaction rate (3.3) up to the cyclic ratio *n* (2.1), as follows:

$$a(n) = a_p(n) + a_i(n) =$$
$$p\cdot n + \int_0^n \frac{\mathrm{d}a_i(n)}{\mathrm{d}n}dn$$
$$= (p-i)\cdot n - i\cdot\ln(1-n) \qquad (3.4)$$

The overall increase of FCG *a*(*n*) consists of the initial growth a_p (3.1) and of the integral of interaction rate (3.3) as:

$$a(n) = a_p(n) + a_i(n)$$
$$= p\cdot n + \int_0^n \frac{\mathrm{d}a_i(n)}{\mathrm{d}n}dn$$
$$= (p-i)\cdot n - i\cdot\ln(1-n) \qquad (3.5)$$

The fatigue crack size curve $a(N,\Delta\sigma)$ under applied cyclic load of stress range $\Delta\sigma$ for critical number N_f of failures, also accounting for possible starting crack size a_0(*n*=0) is then as shown:

$$a(N,\Delta\sigma) = a_0 + N_f(\Delta\sigma)\cdot a(n) \qquad (3.6)$$

Parameter *p* is the propensity to fatigue yielding at the beginning of cyclic loading. The parameter *i*=*p*/*r* is the interaction intensity of FCG and REF (Figure 2). The integration of (3.5) provides the specific energy absorption per cycle under cyclic stresses of constant range $\Delta\sigma$ (for example in *J*/(*N* x cycles), as shown:

$$u(n) = \int_0^n a(n)dn = (p-i)\cdot n^2/2$$
$$+i\cdot[n+(1-n)\cdot\ln(1-n)] \qquad (3.7)$$

The total energy $U(N,\Delta\sigma)$ of resistance to fatigue cracking using (3.6) (for example in *J* x cycles /*N* is then as shown:

$$U(N,\Delta\sigma) = N_f^2(\Delta\sigma)\cdot u(n) \qquad (3.8)$$

Links Between Interaction Model of FCG and Fracture Mechanics

FCG tests under cyclic load of selected stress range $\Delta\sigma_t$ provide the crack size $a_t(N_t)$ for the number of load cycles N_t. The Paris-Erdogan's (PE) power rule [8-10] defines the crack growth rate of the steady FCG regime for $a_t(N_t)>a_s$ (also denoted as 'region two' of FCG starting at crack size as), by two parameters *m* and *C* (the slope and the intercept of the stress intensity factor (SIF or K) range fitted to straight line in the logarithmic scale, as follows:

$$\frac{\mathrm{d}a_t}{dN_t} = C\cdot\Delta K_t^{\ m}(a_t) \qquad (4.1)$$

Irwin's stress intensity factor (SIF or K) range [11] at the end of a crack in Linear Elastic Fracture Mechanics (LEFM) for the measured crack size at using the joint geometry function $Y(a_t)$ as a correction for limited crack growth, is:

$$\Delta K_t(a_t) = \Delta\sigma_t\cdot Y(a_t)\cdot\sqrt{\pi a_t} \qquad (4.2)$$

The record of fatigue test crack size curve is employed for determination of the propensity p and the intensity *i* of the FCEI model curve $a_t(N_t, \Delta\sigma_t)$ (3.6) in region two of FCG according to (3.1-3.8) as shown:

$$a_t(N,\Delta\sigma_t) = a_0 + N_a(\Delta\sigma_t)$$
$$\cdot a_t(n) = a_0 + N_a(\Delta\sigma_t)$$
$$[(p-i)\cdot n - i\cdot\ln(1-n)] \qquad (4.3)$$

The experimentally defined FCG test curves $a(N,\Delta\sigma_t)$ for test stress range $\Delta\sigma_t$ described (4.1-4.3) can be recalculated to other arbitrary stress ranges $\Delta\sigma$.

By substituting the FCG rate *da*/*dN* (3.3) for the FCG curve at (4.1) into the (PE) power rule equation (4.1) the parameters *C* and *m* became attainable from the following relation:

$$\frac{\mathrm{d}a_t}{\mathrm{d}N_t} = p + i\cdot\frac{n_t}{1-n_t} = C\cdot\Delta K_t^{\ m}(a_t) \qquad (4.4)$$

Links Between Fracture Mechanics and Fatigue Life

The stress-life analytic (S-N curves) procedure [12] for fatigue life prediction in engineering of materials for steady FCG regime and high cycle low-strain where the nominal strains are elastic is commonly based on Basquin's type equation [13] derived from the Hooke's law in a form of a power rule as follows:

$$S^n\cdot N = A \qquad (5.1)$$

In (5.1) *S* is the applied stress range *Δσ* or often the stress amplitude *Δσ*/2, *N* is the number of cycles to failure or to transition from steady state to unsteady FCG regime, *n* is the slope (Basquin constant) and *A* is the intercept of the S-N curve with the N axes in logarithmic scale.

The hypothesis of the next study of fatigue life is that the specific energy absorption per load cycle u_t (3.6) during of steady FCG under applied test stress range $\Delta\sigma_t$ is constant and can be analytically related to energy absorption *U*(*N*, *Δσ*) (3.7) under calculational cyclic loads with other stress range *Δσ*. The energy absorption $U(N_t, \Delta\sigma_t)$ of the FCEI model i.e. the energy of resistance to cracking (3.7) equals to the work done on crack growth $W(N_t, \Delta\sigma_t)$ that can be calculated through numerical integration of experimentally derived FCG curves $a_t(N_t)$ (Example, Figure 3) as shown:

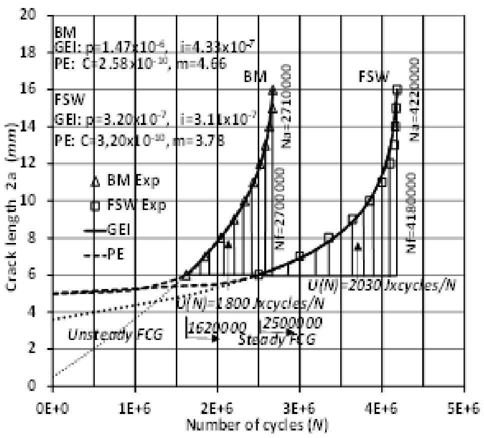

Figure 3: FCG size a(*N*).

$$U(N_t, \Delta\sigma_t) = W(N_t, \Delta\sigma_t)$$
$$= N_t^2(\Delta\sigma_t)\cdot u_t(n_t) \qquad (5.2)$$

The specific absorbed energy $u_t(n_t)$ in (5.2) (for example in *J* / (*N* x cycles) can be calculated according to (3.7) during steady FCG after $n_s = N_s/N_a$ cycles at crack size as, as follows:

$$u_t(n_t) = \int_{n_s}^{n_t} a_t(n)dn =$$
$$(p-i)\cdot n_t^2/2 + i\cdot$$
$$\left[n_t + (1-n_t)\cdot \ln(1-n_t)\right] - u_t(n_s) \quad (5.3)$$

The article investigates in the sequel the possibility for determination and verification of S-N curve parameters *n* and *A* in (5.1) directly from the energy absorption u_t (5.3) during testing a steady FCG regime under test stress $\Delta\sigma_t$ (5.2).

The fatigue lifetime analysis requires several tests with different stress ranges *Δσ* to recriate the S-N curve shape of Basquin's equation (5.1).

The energy *U*(*N*, *Δσ*) absorbed at any selected cyclic stress range *Δσ* can be recalculated to crack size *a* equal to the recorded crack size $a = a_t$ with respect to the energy $U(N_t, \Delta\sigma_t)$ equal to the work done $W(N_t, \Delta\sigma_t)$ until N_t numbers of cycles to failure (3.9) under testing stress range $\Delta\sigma_t$ following the principles of fracture mechanics (4.1-4.4) as shown:

$$U(N,\Delta\sigma) = \int_{N_s}^{N} a(N)dN =$$
$$\left(\frac{\Delta\sigma_t}{\Delta\sigma}\right)^m \cdot \left(\frac{\Delta\sigma_t}{\Delta\sigma}\right)^2 \cdot$$
$$\int_{N_s}^{N}\left(\frac{Y(a_t)}{\Delta K(a_t)}\cdot\frac{\Delta K(a)}{Y(a)}\right)^2 \cdot a_t$$
$$(N,\Delta\sigma_t)dN - U(N_s,\Delta\sigma) \qquad (5.4)$$

Scaling factor $f_a(\Delta\sigma)$ representing the effects of crack size on crack growth for finite sheet according to laws of fracture mechanics with respect to energy absorption comes from (5.4) for each applied stress range as follows:

$$f_a^2(\Delta\sigma)\cdot\int_{N_s}^{N} a_t(N,\Delta\sigma_t)dN =$$
$$\int_{N_s}^{N}\left(\frac{Y(a_t)}{\Delta K(a_t)}\cdot\frac{\Delta K(a)}{Y(a)}\right)^2 \cdot a_t(N,\Delta\sigma_t)dN \quad (5.5)$$

Following (5.5) the (5.4) then can be rewritten using (5.2) and the scaling factor fa as shown:

$$U(N,\Delta\sigma) = \int_{N_s}^{N} a(N)dN =$$
$$\left[\left(\frac{\Delta\sigma_t}{\Delta\sigma}\right)^{\frac{m}{2}+1}\cdot f_a(\Delta\sigma)\right]^2 \cdot N_t^2(\Delta\sigma_t)\cdot u_t(n) \quad (5.6)$$

On the other hand, from the GEI model (3.1-3.7) by definition follows the energy of resistance to crack growth under stress range *Δσ* for appropriate number of cycles to failure *N* as shown:

$$U(N,\Delta\sigma) = \int_{N_s}^{N} a(N)dN$$
$$= N^2(\Delta\sigma)\cdot u_t(n) \qquad (5.7)$$

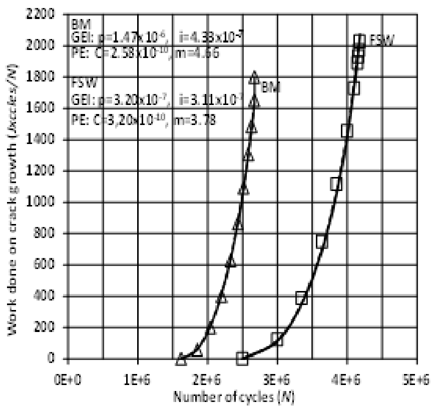

Figure 4: Energy absorption during FCG..

The two relations between energy absorption at optional loading *Δσ* (5.7) and at test loading $\Delta\sigma_t$ (5.6) (Figure 4) provide the term in a form of corrected Basquin's equation as shown:

$$\frac{N}{N_t} = \left(\frac{\Delta\sigma_t}{\Delta\sigma}\right)^{\frac{m}{2}+1}\cdot f_a(\Delta\sigma) \qquad (5.8)$$

For infinite sheet with constant geometry function *Y*(*a*) and for critical SIF K_{cr} is $f_a = 1$ and the above relation (5.8) can be rewritten in the form of standard S-N curve format using stress range values *Δσ* for *S* in (5.1) as follows:

$$\Delta\sigma^n \cdot N = S^n \cdot N = A \qquad (5.9)$$

The value of the S-N curve slope n (Basquin's constant) in (5.9) for steady FCG regime is a simple linear function of the slope m of the SIF range curve in (4.2) as presented below:

$$n = \frac{m}{2} + 1 \qquad (5.10)$$

The intercept A of the S-N curve with the N-axis in (3.5) follows from test data for $\Delta\sigma_t$ and N_t and appropriate value of FCG rate (4.2) characteristic for the transition from steady to unsteady FCG.

For finite sheet w nonlinear geometry function $Y(a)$ and for critical SIF K_{cr} in (5.8) the scaling factor $f_a(\Delta\sigma)$ depends on the effect of crack size on crack growth $Y(a_t)/Y(a)$ for tested $\Delta\sigma_t$ and applied $\Delta\sigma$ stress ranges.

For variable joint geometry functions Y the relation (5.8) can be corrected by finding c from the following condition:

$$\left(\frac{\Delta\sigma_t}{\Delta\sigma}\right)^{\frac{m}{2}+c} = \left(\frac{\Delta\sigma_t}{\Delta\sigma}\right)^{\frac{m}{2}+1} \cdot f_a(\Delta\sigma) \qquad (5.11)$$

For variable scaling factor $f_a(\Delta\sigma)$ the slope n in (5.11) is modified for different stress ranges $\Delta\sigma$ as follows:

$$n = \frac{m}{2} + c \qquad (5.12)$$

The correction c in (5.12) implies the changes caused by effect of crack size to crack growth as shown:

$$c = 1 + \frac{\log\left[f_a(\Delta\sigma)\right]}{\log(\Delta\sigma_t / \Delta\sigma)} \qquad (5.13)$$

For variable scaling factor $f_a(\Delta\sigma)$ averaging methods are required to bring the S-N curves to standard linear form in logarithmic scale (5.1).

The necessary information for the prediction of full lifetime, including unstable crack growth (region three) beyond the steady FCG regime (region two), is the number of cycles to total failure N_{ft} determined under test load $\Delta\sigma_t$.

The simple method for lifetime prediction till total failure under limited information on unsteady crack growth (region three) is the extrapolation of (5.9) for the number of cycles to total failure N_{ft} determined under test load $\Delta\sigma_t$ as follows

$$\Delta\sigma_t^n \cdot N_{ft} = S^n \cdot N_{ft} = A_f \qquad (5.14)$$

Example

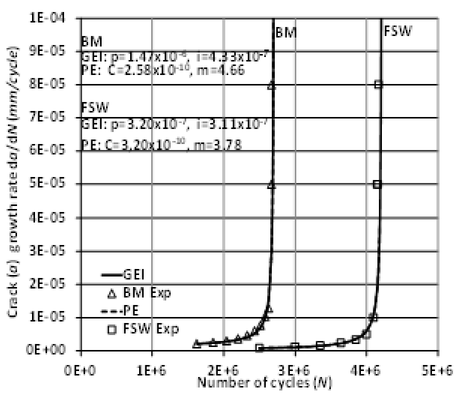

Figure 5: FCG rate da(N)/dN.

In this section, the appropriateness of the FCEI model is demonstrated by examples of FCG tests reported for Base Material (BM) and Friction Stir Welded joints (FSW) of AISI 409M grade ferritic stainless-steel joints [14] (Figures 3-8). The results of the example are summarized in the figures. Examples confirmed the agreement of calculated and reported data.

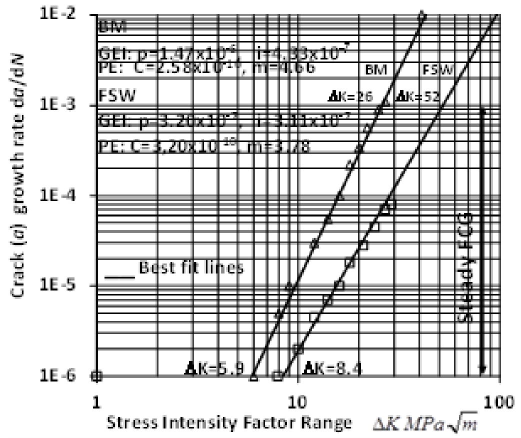

Figure 6: Stress intensity factor ΔK(da/dN).

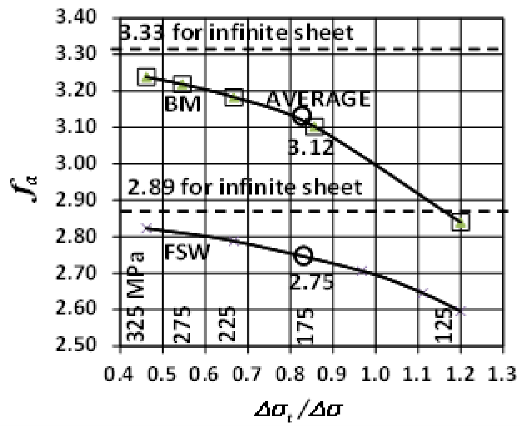

Figure 7: Factor f_s for crack size effect.

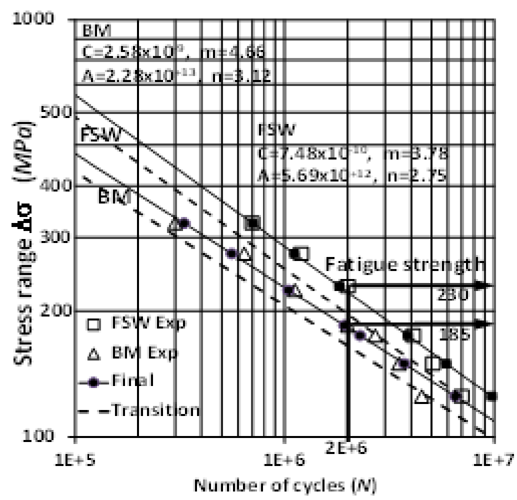

Figure 8: S-N curves.

Conclusion

The review shows that the macroscopically observable fatigue crack growth can be modeled as a mechanical yielding process induced by the redistribution of loads and overstressing among huge but finite numbers of failed and intact internal microstructural bonds in materials. Consequently, fatigue crack growth is regarded as an interaction between fatigue crack growth and fatigue endurance. The fatigue crack growth-endurance-interaction model makes it possible to assess the parameters of fatigue lifetime predictions directly from fatigue crack growth measurements rather than from sets of stress-life tests of lifetime duration under various loading conditions.

Nomenclature

a: crack size	*N*: number of loading cycles
A: intercept of S-N curves	*P*: propensity to interaction parameter
c: correction of S-N curve slope for crack size	*R*: stress ratio
C: intercept of SIF range curves, cause	*R*: factor of fatigue endurance
e: fatigue endurance	*S*: sensitivity to cracking
E: effect of C	*S*: stress range or amplitude in S-N approach
f_a: scaling factor	*Y*: joint geometric function
I: interaction intensity parameter	*W*: specimen width
M: slope of SIF range curves	ΔK: Stress Intensity Factor (SIF) range
N: slope of S-N curves, also cyclic ratio	$\Delta\sigma$: cyclic stress range

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