

An Iterative Method for Solving the Problem of Diffraction of Longitudinal Shear Waves on A System of Thin Rigid Inclusions


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Abstract

The problem of the diffraction field determination is arising as a result of the longitudinal shear wave interaction with the thin rigid inclusions system arbitrarily situated in an infinity body was solved. Inclusions are considered to be fully coupled to the elastic medium and are moving. Unknown amplitudes of inclusions are determined from the equations of motion. The solution method is based on the submission diffraction field displacement as sum of discontinuous solutions to the Helmholtz equation, the constructed for each inclusion. As result the original problem is reduced to the system of the singular integral equations for un-known jumps of stresses on the inclusions surface, The iterative method of this system solving, where the zero approximation are the solutions of the integral equations for the single inclusions, is proposed. This integral equation for single inclusions are numerical solved the mechanical quadrature method. The final result is the approximate formulas for calculating stress intensity factors and the amplitudes of the oscillations.

Keywords: Thin ridged inclusion; Wave interaction; Integral equations; Iterative method First

Introduction

Nowadays methods of mechanics of deformable bodies (potentials method, discontinuous solutions method, etc.) make it easy to reduce the problem of determination of wave fields in a body with arbitrary numbers system of thin inclusions or cracks to the solution systems of integral equations. But as shows the analysis of the scientific publications, the main attention is paid to bodies with systems of cracks. Research of dynamic stress state bodies with systems of thin rigid inclusions is much less. One of the first works in this direction, probably should be considered [1], where is solved the problem of diffraction of waves on two parallel thin rigid inclusions. Interaction of waves with periodic systems of curvilinear inclusions is considered in [2]. The method of explosive articles decisions Wave fields and Stress Intensity Factors (SIF) for elastic waves diffraction on systems of radially placed inclusions are defined of discontinuous solutions method in [3-5]. Achievements in solving dynamic problems for bodies with arbitrary systems of inclusions are associated with the development of for boundary integral equations and boundary elements methods can be seen in [6,7]. But when applying methods, based on the reducing the original boundary value problems for bodies with inclusions to systems of integral equations, there is a significant problem. It consists in the need to solve systems of integral equations of large dimension. That's why during calculation of SIF and wave fields, authors of the limited case of a few, often just two, inclusions. Thus, the determination of dynamic stress state and wave fields in bodies with systems of thin rigid inclusions is not completely solved. In the submitted article, iterative method for determination of SIF and wave fields in the interaction of the longitudinal shear waves of N inclusions. As a result, there is no need to solve a system of N integral equations, and the N single equations are solving on each iteration. These equations correspond to a single inclusion and differ only in the right side.

Statement of the Problem

Let an elastic body that is in a longitudinal shear strain condition, N through thin rigid inclusions are situated. These inclusions are occupied on the plane Oxy by the segment's length $2d_k$ and with centers at points with coordinates $(a_k, b_k), k=1, 2, \dots, N$ (Figure 1).

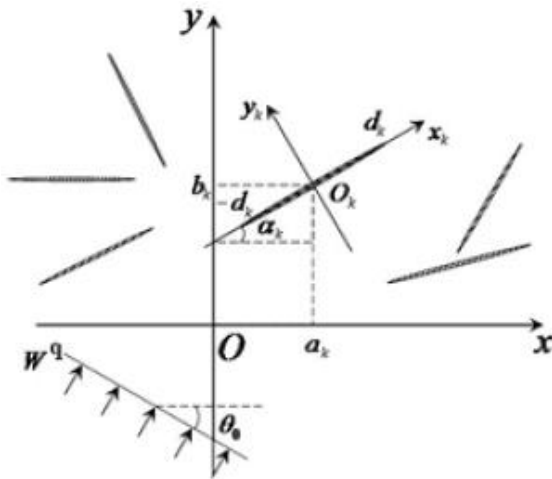


Figure 1: The system of the thin ridged inclusions in the body.

The wave of longitudinal shear interacts with them which is propagated in the body and causing the following displacement along axis Oz :

$$W^q(x, y) = Ae^{ik_2(x \cos \theta_0 + y \sin \theta_0)}, k_2^2 = \frac{\rho \omega^2}{G} \tag{1}$$

Where ω - the frequency of the oscillations, ρ, G - density and modulus of shear of the body, θ_0 - the angle between the direction of propagation of the wave and the positive direction of the axis O_x . Multiplier $e^{-i\omega t}$ which determines dependency from time here and then rejected. Let $W(x, y)$ is the only different from 0 in the conditions of longitudinal shear strain component of the vector of displacements caused by waves reflected from the defects. Then it must satisfy the Helmholtz equation

$$\Delta W + k_2^2 W = 0 \tag{2}$$

where Δ - the Laplace operator in a system of coordinates Oxy . This equation must be seen with conditions on the inclusions for the formulation that the local coordinate systems $O_k x_k y_k$ connected with each of them. The relationship between the global and the local coordinate systems as well as between local coordinate systems connected with a variety of inclusions are given formulas

$$\begin{cases} x = a_k + x_k \cos \alpha_k - y_k \sin \alpha_k, \\ y = b_k + x_k \sin \alpha_k + y_k \cos \alpha_k \end{cases} \tag{3}$$

$$\begin{cases} x_l = (a_k - a_l) \cos \alpha_l + (b_k - b_l) \sin \alpha_l + x_l \cos(\alpha_k - \alpha_l) - y_l \sin(\alpha_k - \alpha_l) \\ y_l = -(a_k - a_l) \sin \alpha_l + (b_k - b_l) \cos \alpha_l + x_l \sin(\alpha_k - \alpha_l) + y_l \cos(\alpha_k - \alpha_l) \end{cases} \tag{3}$$

$$l, k = 1, 2, 3, \dots, N$$

It is assumed that inclusion is completely coupled with an elastic medium that leads to equation

$$W_k(x_k, 0) = c_k - W_k^q(x_k, 0), W_k^q(x_k, 0) = -Ae^{ik_2 z_{0k}} \tag{4}$$

$$z_{0k} = a_k \cos \theta_0 + b_k \sin \theta_0 + x_k \cos(\alpha_k - \theta_0) - y_k \sin(\alpha_k - \theta_0), k=1, 2, 3, \dots, N$$

Also, the surface impurities cause a discontinuity stress unknown jumping which denomination

$$\tau_{zy_k}(x_k, +0) - \tau_{zy_k}(x_k, -0) = \chi_{1k}(x_k), -d_k < x_k < d_k, k=1, 2, \dots, N \tag{5}$$

In equality (4) consists of k c - unknown displacement inclusions under the influence of propagating wave. They are determined from the equations of motion of inclusions, which in the case of harmonic oscillation are of the form

$$m_k \omega^2 c_k = \int_{-d_k}^{d_k} \chi_{1k}(x_k) dx_k, k=1, 2, \dots, N \tag{6}$$

where m_k - mass of k -th inclusion.

When formulated conditions the problem of determination the displacement and stress of the diffraction field in the body and SIF for inclusions is posed.

Solution of the Problem

For the solution of the problem for each inclusion with a number l in a coordinate system $O_l x_l y_l$ connected with it, of discontinuous solutions of equation (2) [8,9] from jumping (5) are built

$$W_l^d(x_l, y_l) = \int_{-d_l}^{d_l} \frac{\chi_{1l}(\eta)}{G} r_2(\eta - x_l, y_l) d\eta, l=1, 2, \dots, N \tag{7}$$

Displacement of the scattered wave field in the coordinate system Oxy is represented in the form of

$$W(x, y) = \sum_{l=1}^N W_l^d(x, y), \tag{8}$$

Where $W_l^d(x, y)$ are obtained from (7) as a result of the first coordinate transformation formulas (3) for $k=l$. But in order to use the formulas (8) it is necessary to determine the unknown jumps of stress (5) on the inclusions. For determination of the unknown jumps $\chi_{1l}(x_l)$ from the conditions (4) the system of integral equations must be obtained. That the received system has received singularity Cauchy type, (4) is replaced by the following equivalent equalities.

$$\frac{\partial W_k}{\partial x_k}(x_k, 0) = -\frac{\partial W_k^q}{\partial x_k}(x_k, 0), W_k(-d_k, 0) = c_k - W_k^q(-d_k, 0), k=1, 2, \dots, N \tag{9}$$

The first equality (9) is the result of applying to (4) the operation of differentiation, and the second equality is equivalence of differentiated and initial equalities. After transformations of coordinates in the (8) for other formulas (3) and substitution to (9)

the following system of singular integral equations with additional conditions to determine the jumps are obtained:

$$\frac{1}{2\pi} \int_{-1}^1 \varphi_k(\tau) \left(-\frac{1}{\tau-\zeta} + F_k(\tau-\zeta) \right) d\tau + \sum_{l=1}^N \frac{1}{2\pi} \int_{-1}^1 \varphi_l(\tau) R_{kl}(\tau, \zeta) d\tau = -ik_0 A_0 \cos(\alpha_k - \theta_0) e^{ik_0 \rho_k(\zeta)},$$

$$\frac{\gamma_k}{2\pi} \int_{-1}^1 \varphi_k(\tau) [\ln(1+\tau) + D_k(\tau+1)] d\tau + \sum_{l=1}^N \frac{\gamma_k}{2\pi} \int_{-1}^1 \varphi_l(\tau) D_{kl}(\tau) d\tau = c_{0k} - A_0 e^{ik_0 \rho_k(-1)}, k=1, 2, \dots, N \quad (10)$$

In this system, introduced the notation

$$\varphi_k(\tau) = \frac{\chi_k(d, \tau)}{G}, k=1, 2, \dots, N; \eta = d_1 \tau, x_k = d_k \zeta, \gamma_k = \frac{d_k}{d}, \delta_k = \frac{a_k}{d}, \varepsilon_k = \frac{b_k}{d}, \quad (11)$$

$$k_0 = k_2 d, d = \max(d_1, d_2, \dots, d_N)$$

Kernels of integral operators and the right-side equations (10) are determined by the formulas.

$$F_k(x) = o(x \ln|x|), D_k(x) = o(x^2 \ln|x|), x \rightarrow 0,$$

$$R_{kl}(\tau, \zeta) = -\frac{ik_0 \gamma_l}{2} \frac{H_1^{(1)}(k_0 q_{kl})}{q_{kl}^0} ((\gamma_l \tau - a_{kl}) \cos \alpha_{kl} - (\gamma_k \zeta + b_{kl}) \sin \alpha_{kl}),$$

$$D_{kl}(\tau) = -\frac{i\pi}{2} H_0^{(1)}(k_0 h_{kl}(\tau)), h_{kl}^2 = (\gamma_l \tau - a_{kl} + \gamma_k \cos \alpha_{kl})^2 + (b_{kl} - \gamma_k \sin \alpha_{kl})^2,$$

$$a_{kl} = (\varepsilon_k - \varepsilon_l) \cos \alpha_l + (\delta_k - \delta_l) \sin \alpha_l, b_{kl} = -(\varepsilon_k - \varepsilon_l) \sin \alpha_l + (\delta_k - \delta_l) \cos \alpha_l,$$

$$r_{0k}(\zeta) = a_k^{(i)} + \gamma_k \zeta \cos(\alpha_k - \theta_0), a_k^{(i)} = \varepsilon_k \cos(\theta_0) + \delta_k \sin(\theta_0)$$

To the system (10) we must also add the equality (6) to determine the unknown amplitudes of oscillation of inclusions in the notation (11) take the form

$$c_{0k} = -\frac{1}{2\gamma_k h_{0k} k_0^2 \rho_{k-1}} \int_{-1}^1 \varphi_k(\tau) d\tau, c_{0k} = \frac{c_k}{d}, \overline{\rho}_k = \frac{\rho_k}{\rho}, h_{0k} = \frac{h_k}{d_k} \quad (12)$$

where h_k, ρ_k are the thickness and density of the material of k^{th} inclusion.

Iterative Method Solution

The system of integral equations (10), (12) can be solved approximately using the by the mechanical quadrature method, as in [3-5]. But the immediate use of this method requires solving a system of linear algebraic equations, which is proportional to the number of inclusions N. Therefore, the iterative method for solving the system (12) is proposed for elimination of this problem. For this, the zero approximation is

solution of N separate equations for single inclusion

$$\frac{1}{2\pi} \int_{-1}^1 \varphi_k^{(0)}(\tau) \left(-\frac{1}{\tau-\zeta} + F_k(\tau-\zeta) \right) d\tau = -ik_0 A_0 \cos(\alpha_k - \theta_0) e^{ik_0 \rho_k(\zeta)}$$

$$\frac{\gamma}{2\pi} \int_{-1}^1 \varphi_k^{(0)}(\tau) [\ln(1+\tau) + D_k(\tau+1)] d\tau = c_{0k} - A_0 e^{-ik_0 \rho_k(-1)} \quad (13)$$

$$c_{0k}^{(0)} = -\frac{1}{2\gamma_k h_{0k} k_0^2} \int_{-1}^1 \varphi_k^{(0)}(\tau) d\tau, k=1, 2, \dots, N$$

Then an iterative process continues by solving the next equations at every step

$$\frac{1}{2\pi} \int_{-1}^1 \varphi_k^{(i)}(\tau) \left(-\frac{1}{\tau-\zeta} + F_k(\tau-\zeta) \right) d\tau = -ik_0 A_0 \cos(\alpha_k - \theta_0) e^{ik_0 \rho_k(\zeta)}$$

$$- \sum_{l=1}^N \frac{\gamma_l}{2\pi} \int_{-1}^1 \varphi_l^{(i-1)}(\tau) R_{kl}(\tau, \zeta) d\tau; \quad (14)$$

$$\frac{\gamma_k}{2\pi} \int_{-1}^1 \varphi_k^{(i)}(\tau) [\ln(1+\tau) + D_k(\tau+1)] d\tau = c_{0k}^{(i)} - A_0 e^{-ik_0 \rho_k(-1)}$$

$$- \sum_{l=1}^N \frac{\gamma_l}{2\pi} \int_{-1}^1 \varphi_l^{(i-1)}(\tau) D_{kl}(\tau, \zeta) d\tau;$$

$$c_{0k}^{(i)} = -\frac{1}{2\gamma_k h_{0k} k_0^2} \int_{-1}^1 \varphi_k^{(i)}(\tau) d\tau, i=1, 2, 3, \dots$$

Solution of integral equations (13), (14) is based on the representation of unknown functions in the form.

$$\varphi_k^{(i)}(\tau) = \frac{\psi_k^{(i)}(\tau)}{\sqrt{1-\tau^2}}, k=1, 2, \dots, N, i=1, 2, \dots \quad (15)$$

and approximation of functions of the interpolation polynomial

$$\psi_k^{(i)}(\tau) \approx \sum_{m=1}^N \psi_{km}^{(i)} \frac{T_n(\tau)}{(\tau - \tau_m) T_n'(\tau_m)}; \psi_{km}^{(i)} = \psi_k^{(i)}(\tau_m) \quad (16)$$

where $T_n(\tau)$ is the Chebyshev polynomial, τ_m are the roots of this polynomial. The formula (15), (16) provide an opportunity to numerically solve equations (13), (14) method of mechanical quadrature's using Chebyshev polynomials $U_{n-1}(\zeta)$: roots

$$\zeta_j = \cos \frac{\pi j}{n}, j=1, 2, \dots, n-1$$

as collocation points. When applying this method known quadrature formula [10] is used for Cauchy integrals and Gauss-Chebyshev f quadrature formula [11] is used for regular integrals. To the integral with logarithmic singularity the formula obtained [3,4,9] is applied.

$$\int_{-1}^1 \varphi_k^{(i)}(\tau) \ln(1+\tau) d\tau = \sum_{m=1}^n a_m B_m \psi_{km}^{(i)}; B_m = -\ln 2 - 2 \sum_{p=1}^{n-1} \frac{(-1)^p}{p} \cos \frac{p(2m-1)\pi}{2n}$$

As a result, equations (14) are reduced to the solution of systems of linear algebraic equations sequence that differ only in the right sides of

$$\frac{1}{2\pi} \sum_{m=1}^n a_m \psi_{km}^{(0)} \left(-\frac{1}{\tau_m - \zeta_j} + F_k(\tau_m - \zeta_j) \right) = -ik_0 A_0 \cos(\alpha_k - \theta_0) e^{ik_0 \rho_k(\zeta_j)};$$

$$\frac{\lambda_k}{2\pi} \sum_{m=1}^n a_m \psi_{km}^{(0)} (B_m + D_k(\tau_m + 1)) = c_{0k}^{(0)} - A_0 e^{ik_0 \rho_k(-1)},$$

$$\frac{\lambda_k}{2\pi} \sum_{m=1}^n a_m \psi_{km}^{(0)} (B_m + D_k(\tau_m + 1)) = c_{0k}^{(0)} - A_0 e^{ik_0 \rho_k(-1)}, \quad (17)$$

$$c_{0k}^{(0)} = -\frac{1}{2\gamma_k h_{0k} k_0^2} \sum_{m=1}^n a_m \psi_{km}^{(0)},$$

$$\frac{1}{2\pi} \sum_{m=1}^n a_m \psi_{km}^{(i)} \left(-\frac{1}{\tau_m - \zeta_j} + F_k(\tau_m - \zeta_j) \right) = -ik_0 A_0 \cos(\alpha_k - \theta_0) e^{ik_0 \zeta_j} - \sum_{\substack{l=1 \\ l \neq k}}^N \frac{1}{2\pi} \sum_{m=1}^n a_m \psi_{lm}^{(i)} F_{kl}(\tau_m, \zeta_j)$$

$$\frac{\gamma_k}{2\pi} \sum_{m=1}^n a_m \psi_{km}^{(i)} (B_m + D_k(\tau_m + 1)) = c_{0k}^{(i)} - A_0 e^{ik_0 \zeta_j (-1)} - \sum_{\substack{l=1 \\ l \neq k}}^N \frac{\gamma_l}{2\pi} \sum_{m=1}^n a_m \psi_{lm}^{(i)} D_{kl}(\tau_m),$$

$$c_{0k}^{(i)} = -\frac{1}{2\gamma_k h_{0k} k_0^2} \sum_{m=1}^n a_m \psi_{km}^{(i)}, k=1, 2, \dots, N; j=1, 2, \dots, n-1; i=1, 2, 3, \dots$$

After solving this system functions $\psi_k^i(\tau)$ are approaching interpolation polynomials (16). As it is known [12] stresses in the body around the thin rigid inclusions are defined by asymptotic formulas.

$$\begin{bmatrix} \tau_{xy} \\ \tau_{xy} \end{bmatrix} = -\frac{k_l^\pm}{\sqrt{r}} \begin{bmatrix} \sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} \end{bmatrix} + o(1), r > 0, l=1, 2, \dots, N,$$

where (r, θ) are polar coordinates with the pole in right or left end of the inclusion. The coefficients k_l^\pm are usually called Stress Intensity Factors (SIF) for inclusions. After solving the system (17) their approximate value by results of the i^{th} iteration are calculated on formulas

$$k_l^\pm = \frac{k_l^\pm}{G_l \sqrt{2d_l}} = \psi_l^{(i)}(\pm 1), l=1, 2, \dots, N \tag{18}$$

$$\psi_l^{(i)}(+1) = \frac{1}{n} \sum_{m=1}^n (-1)^{m+1} \psi_{lm}^{(i)} \operatorname{ctg} \left(\frac{\beta_m}{2} \right), \psi_l^{(i)}(-1) = \frac{1}{n} \sum_{m=1}^n (-1)^{n-m} \psi_{lm}^{(i)} \operatorname{tg} \left(\frac{\beta_m}{2} \right),$$

$$\beta_m = \frac{\pi(2m-1)}{2n}$$

Results of Numerical Analyses

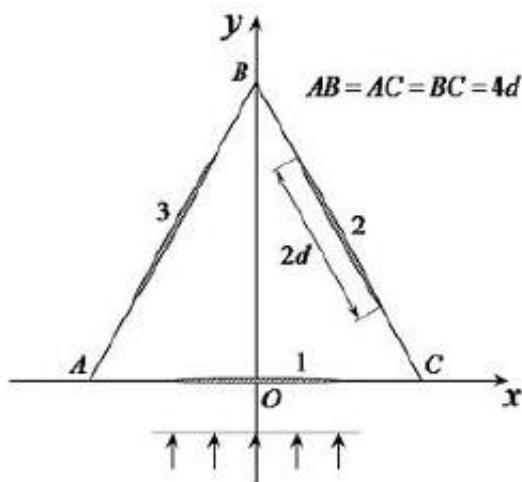


Figure 2: The system of the three inclusions.

One of the main purposes of the numerical analysis was to study practical convergence of iterative method is proposed. This was a system with three inclusions of the same length $2d$ is considered, Inclusion is placed at the sides of the proper triangle with sides long $4d$, as shown in Figure 2 The thickness and density of the inclusions [13].

There are considered to be the same: $h_{0k} = 0.05, \overline{\rho_{0k}} = 1$. It was assumed that plane wave of longitudinal shear, propagating along the positive direction of the axis $Oy(\theta_0 = 90^\circ)$ interacts with inclusions. The Figure 3 shows the dependency graphs of absolute value of the dimensionless SIF $|k_l| = |k_l^-| = |k_l^+|$ for first inclusion of the dimensionless wave number $k_0 = k_2 d$. Curve 1 corresponds to the values SIF obtained as a result of the immediate solution of the system of integral equations (14) mechanical quadrature method. Curves 2,4 show the value of the SIF is obtained by the formula (18) as a result of the implementation of the specified number of iterations. We can see that after 4 iterations the results are obtained by different methods almost do not differ. The continuation of the iterative process lids to the complete coincidence of the results. And at the frequencies of oscillations such that convergence results is observed already after two iterations. Further the interaction of such the waves with system of 7 inclusions Figure 4 is considered. This system consists of inclusion at each end of which 3 radially inclusion, which are three times less length are placed.

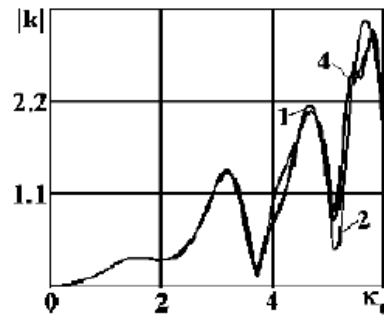


Figure 3: Practical convergence of iterative method.

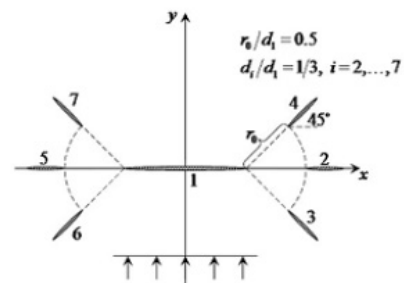


Figure 4: The system of the three inclusions.

Results of calculation of SIF are shown in graphs in Figures 5 & 6. Curves 1, 2, 3, 4 in Figure 5 show the change in the absolute values

of the SIF around the nearby ends of inclusions: $|k_1^+|, |k_2^-|, |k_3^-|, |k_4^-|$. We can see that the growth frequency (dimensionless wave number) the monotonous increase SIF is observed for all inclusions i , then the frequency dependence becomes oscillating. The largest value of SIF is observed s for short inclusions inclining to long inclusion. The graphs dependence of the absolute values of the SIF on the dimensionless wave number (dimensionless frequency) around the distant ends of inclusions are shown in Figure 6 Curves with the numbers 2, 3, 4 correspond to the values $|k_2^+|, |k_3^+|, |k_4^+|$. These charts show tendency of increasing SIF to reaching for the greatest value around which oscillation is there. -We can also see that short inclusions SIF around the distant ends significantly exceed SIF around the nearby ends of inclusions. Curves 1-4 on the Figure 7 illustrate the dependence of amplitude oscillations of the appropriate inclusions $|c_{0k}|, k=1,2,3,4$ on dimensionless wave number k_0 .

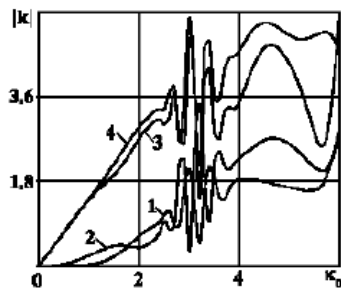


Figure 5: Absolute values of the SIF around the nearby ends of inclusions.

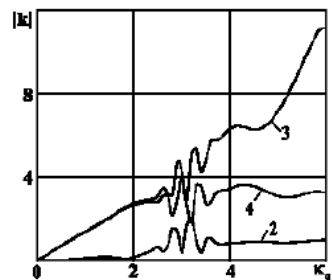


Figure 6: Absolute values of the SIF around the distant ends of inclusions.

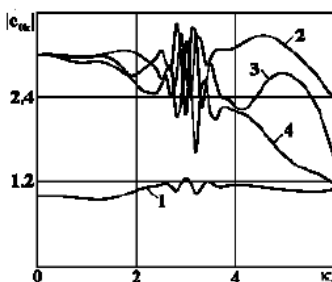


Figure 7: Dependence of amplitude oscillations on dimensionless wave number.

Conclusion

The above results allow making the following general conclusions Effective iterative method of solving the problem by definition stress condition and diffraction fields in body with arbitrary system of thin rigid inclusions at the interaction with the longitudinal shear waves. This method can be used for systems of inclusions with implicate geometry and sufficiently tightly placed inclusions. Numerical investigation has determined that the configuration of a system of inclusions, their size and relative locating substantially influence the value of the SIF. Interaction repeatedly reflected from inclusions of the waves provides dependence to SIF on the frequency of implicate shape with lots of maximums and minimums. On account of the wave interaction short and thus lighter inclusions have larger amplitude of oscillation. For inclusions with larger amplitude of oscillations also and large values of SIF are looked on. The existence of areas with a strong oscillation for frequencies such that satisfied $2.5 \leq k_0 \leq 3.5$ is common for both the SIF and the

amplitude. This fact is due to effect of multiple reflection waves on the surface of inclusions at these frequencies.

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