



Determination of the Probability Size Distribution of Solid Particles in a Technical Water



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Abstract

Measuring the size distribution of solid contaminants born by the technical cooling water has an important role. Therefore, in this paper a partial differential equation which describes the probability size distribution of solid contaminants born by technical cooling water is presented and solved. It is demonstrated that the differential equation possesses an analytical solution in the form of the Gibbs-Boltzmann distribution function. The fitting function of probability density function represents a solution of the partial differential equation.

Before evaluation, the sampled water was prepared with filtration equipment. Extracing particles are digitized under scanning microscope. Finally, the particle sizes of contaminants are measured by digital image analysis technique, incorporated in a special program.

Keywords: Partial differential equation; Waterborn contaminants; Particle size distribution; Probability density function

Introduction

Heat, which can be used no more, is dissipated in many factories. Since water has a high heat absorption capacity, the heat transport is usually carried out by means of cooling water. This technical water is found not only in cooling systems, but also in radiators, etc. Particles of different sizes appear in such liquids. They also occur in gases, such as air pollution. So not only the experiments [1,2] but also the mathematical modeling [3-5] of different processes in technology and agriculture had been strongly intensified during the last time. An example of these many processes is the counting of particles suspended in a liquid or gaseous medium.

In this paper a diffusion equation which describes the water-born solid contaminants size distribution in a technical cooling water is solved. The solutions are experimentally verified in [1,2].

Diffusion Equation

$y = y(x, t)$ denotes the probability size distribution of waterborn solid contaminants with the particle diameter x and the time t , $t > 0$. Then the differential equation which describes this diffusion process takes the form

$$v(xy_{xx} + y_x) - y_t = 0. \quad (1)$$

V is a known positive constant and represents the drift velocity. For getting an idea of how the solution of this diffusion equation might look like, equation (1) is simplified. Writing the factor of V in the form $(xy_x)_x$ and replacing x by a constant k , (1) becomes

$$vky_{xx} - y_t = 0. \quad (2)$$

This is the simplest diffusion equation and possesses the solution

$$y(x, t) = \frac{1}{2\sqrt{\pi vkt}} \exp\left(-\frac{x^2}{4vkt}\right). \quad (3)$$

Since the coefficients of a differential equation also determine the solution, in (1) this is x , and because (2) is also a parabolic equation as (1), it can be assumed that the solution of (1) also is of the form (3). t exists in (1) and (2) only as a derivative. Therefore, it is obvious that t also appears in the solution of (1) as powers, as in (2). So, a solution of the form

$$y(x, t) = Ct^p e^{ax^n t^q} \quad (4)$$

is supposed. C represents an arbitrary constant. The constants a , n , p and q are unknown real numbers. $a=0$, $n=0$ and $q=0$ are excluded. Inserting (4) into the diffusion equation (1) yields the conditional equation

$$pt^{-1} + aqx^n t^{q-1} = van^2(x^{n-1}t^q + ax^{2n-1}t^{2q}) \quad (5)$$

for the four unknown constants. Since x and t are independent variables, the coefficients of each power of x and t vanish. Equation (5) contains four powers of x as well of t . Starting with the comparison of powers of x it is assumed that at least two powers of x in (5) are equal. This excludes trivial solutions. Thus, the following three cases are thereby left:

Case 1: $n=-1$

Now (5) has the form

$$pt^{-1} + aqt^{q-1}x^{-1} - vat^q x^{-2} - va^2 t^{2q} x^{-3} = 0. \quad (6)$$

Equating the factor of x^{-1} or x^{-2} or x^{-3} to zero yields $a=0$, which is useless.

Case 2: $n=1/2$

In this case (5) is

$$\frac{va}{4} t^q x^{-1/2} + \frac{va^2}{4} t^{2q} - pt^{-1} - aqt^{q-1} x^{1/2} = 0. \quad (7)$$

As in Case 1 the factors of $x^{-1/2}$ and $x^{1/2}$ yield $y=y(t)$.

Case 3: $n=1$

Here (5) becomes

$$vat^q - pt^{-1} + a(vat^{2q} - qt^{q-1})x = 0. \quad (8)$$

Therefore, the coefficient comparison with respect to x provides

$$vat^q - pt^{-1} = 0, \quad (9)$$

$$vat^{2q} - qt^{q-1} = 0, \quad (10)$$

Taking $q=-1$ from (9) implies $p=aV$. Substituting $q=-1$ in (10) the comparison of powers with respect to t yields a $V=-1$. This gives $a=-V^{-1}$ and $p=-1$. So the solution (4) of the partial differential equation (1) is

$$y(x,t) = \frac{C}{t} \exp\left(-\frac{x}{vt}\right). \quad (11)$$

The Normalization Condition

Making use of the normalization condition

$$\int_0^{\infty} y(x,t=1) dx = 1$$

and substituting the solution (11) into this integral yield

$$C = \frac{1}{v}.$$

Thus, the final solution

$$y(x,t) = \frac{1}{vt} \exp\left(-\frac{x}{vt}\right) \quad (12)$$

of the diffusion equation is calculated. The papers [1,2] contain the results of the corresponding experiments in which sizes had been measured with a microscopic morphometric method.

Experimental Results

This chapter lists some important results, which are given in [1]. The solid contaminants, suspended in a technical water was used in Tigar Tire company-Pirot, Serbia [6-8]. On the base of experimental data, the empirical probability density function of particle sizes is established using non-linear data fitting by using the iterative algorithm of Levenberg-Marquardt (9), according the exponential function. The peak value (around 600) of the probability density function arises at $d=0.125\text{mm}$ ($125\mu\text{m}$). However, for $d>0.75\text{mm}$ ($755\mu\text{m}$), its value decreases below 10. The smallest evident particle had equivalent diameter of only 0.036mm ($36\mu\text{m}$). Fitting

the probability density function of the equivalent diameter d of solid contaminants, suspended in a technical water, had given

$$y = y_0 + \frac{1}{v} \exp\left(-\frac{x}{v}\right), \quad (13)$$

$y_0 = \text{constant}$. This function represents a solution (12) with $t=1$.

Remark: Formula (13) represents a Boltzmann distribution resp. Gibbs-Boltzmann distribution.

Conclusion

General purpose of mathematical modeling of the processes and operations in modern agriculture is well known and recognized worldwide. Following the common practice, in the present paper is presented a partial differential equation model describing the lateral probability density function of solid particles in a technical water. The basic goal of approximating an experimentally determined probability density function with an analytic function having appropriate shape is to describe a large amount of empirical data, which are represented by empirical statistical distribution.

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