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Review Article

A Two-Dimensional Approach to The Three-Dimensional Phase Resolving Wave Modeling

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Abstract

A new approach to the three-dimensional modeling based on the analysis of three-dimensional equations of potential waves in the periodic domain is developed. Instead of the 3-D equation for the velocity potential, the 2-D Poisson equation on a surface is used. This equation contains both the first and second vertical derivatives of the potential. It is suggested that the equations be closed by introducing the connection between these derivatives. Finally, the model for 3-D waves includes only surface equations, which dramatically simplifies the modeling as well as provides a substantial acceleration of the calculations and reduces a volume of the memory used. The examples of integration of the equation over long periods, taking into account the input energy and dissipation, proved that such a simplified approach gives quite realistic results. It is typical that a new model runs faster by around two orders than the 3-D model

Keywords: Phase resolving; Wave modeling; Wave development; Wave spectrum; Wind input; Dissipation

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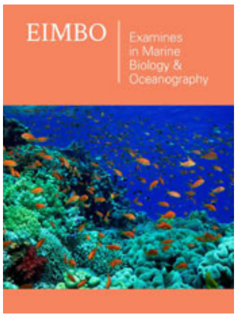

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Introduction

Several different 3-D numerical methods for wave processes investigation have been developed over the past two decades [1]. The most advanced approaches are Boundary Integral Method [2], the finite-difference model designed at Technical University of Denmark [3], High-Order Spectral (HOS) model developed at Ecole Centrale Nantes, LHEEA Laboratory [4], Full Wave Model (FWM), [5-7].

A common disadvantage of the 3-D models is their low performance because all of them somehow or other resolve the vertical structure of wave field on the basis of 3-D equation for the velocity potential. It is well known that most of the computer time for 3-D model is spent on solution of Poisson equation for the velocity potential. However, the 3-D solution is used for calculation of the vertical velocity w on a surface only. A new method is based on separation of the velocity potential φ into the linear $\bar{\varphi}$ and nonlinear $\tilde{\varphi}$ components and on the considering of Poisson equation for the nonlinear component on a surface. This exact equation contains both the first $\left(\tilde{\varphi}_{\zeta} = \tilde{w}\right)$ and second $\left(\tilde{\varphi}_{\zeta\zeta} = \tilde{w}_{\zeta}\right)$ derivatives, which means that the formulation of the entire problem is not closed. The multiple numerical calculations with an accurate 3-D model [8] show that $\tilde{w}$ and $\tilde{w}_{\zeta}$ are well connected. This suggestion was found to be reliable enough to formulate the entire model which can realistically simulate wave dynamics. We call this model Heuristic Wave Model (HWM). The comparison of similar runs with the models supplied by identical physics shows that the new model is by two decimal orders faster than FM.

The model is supplied by a developed system of the run-time processing and recording of the results. The volume of data generated by the model typically exceeds dozens of gigabytes. Note that the given paper gives but a brief description of a new approach. A detailed description of the results will be given in the forthcoming papers.

Derivation of a Simplified 2-D Heuristic Wave Model (HWM) For 3-D Waves

The equations are written in the non-stationary surface-following non-orthogonal coordinate system:

$$\xi = x, \vartheta = y, \zeta = z - \eta\left(\xi, \vartheta, \tau\right), \tau = t \longrightarrow (1)$$

where $\eta(x,y,t)=\eta(\xi,\vartheta,\zeta)$ is a moving periodic wave surface?

The surface conditions for potential waves in the system of coordinates (1) at $\zeta=0$ take the following form:

$$\eta_\tau=-\eta_\xi\varphi_\xi-\eta_\vartheta\varphi_\vartheta+\left(1+\eta_\xi^2+\eta_\vartheta^2\right)\varphi_\zeta \longrightarrow (2)$$

$$\varphi_\tau=-\frac{1}{2}\left(\varphi_\xi^2+\varphi_\vartheta^2-\left(1+\eta_\xi^2+\eta_\vartheta^2\right)\varphi_\zeta^2\right)-\eta-p \longrightarrow (3)$$

Where φ is the velocity potential? The Laplace equation for φ at $\zeta\le 0$ turns into Poisson equation

$$\varphi_{\xi\xi}+\varphi_{\vartheta\vartheta}+\varphi_{\zeta\zeta}=\Upsilon(\varphi)\longrightarrow (4)$$

(4) where Υ is the operator:

$$\Upsilon()=2\eta_\xi()_{\xi\zeta}+2\eta_\vartheta()_{\vartheta\zeta}+\left(\eta_{\xi\xi}+\eta_{\vartheta\vartheta}\right)()_\zeta-\left(\eta_\xi^2+\eta_\vartheta^2\right)()_{\zeta\zeta}\longrightarrow (5)$$

The equations (2)-(5) are written in a non-dimensional form introduced formally by assuming that the gravity acceleration is equal to one.

The equations (2-5) are the basis of the three-dimensional FWM. The method of solution combines a 2-D Fourier transform method in the 'horizontal surfaces' and a second-order finite-difference approximation on the stretched staggered grid that provides high accuracy of a finite-difference approximation in the vicinity of surface. An equation (4) is solved as Poisson equations with the iterations over the right-hand side with the prescribed accuracy.

In was suggested in [5] that it would be convenient to represent the velocity potential as a sum of two components such as the analytical ('linear') component $\bar{\varphi}$ and the arbitrary ('non-linear') component $\tilde{\varphi}$

$$\varphi=\bar{\varphi}+\tilde{\varphi}\longrightarrow (6)$$

The analytical component $\bar{\varphi}$ satisfies Laplace equation:

$$\bar{\varphi}_{\xi\xi}+\bar{\varphi}_{\vartheta\vartheta}+\bar{\varphi}_{\zeta\zeta}=0\longrightarrow (7)$$

(7) with a known solution that satisfies the following boundary conditions:

$$\begin{matrix}\zeta=0:\varphi_0=\varphi\\ \zeta\to-\infty:\bar{\varphi}_\zeta\to 0\end{matrix}\longrightarrow (8)$$

The nonlinear component satisfies an equation:

$$\tilde{\varphi}_{\xi\xi}+\tilde{\varphi}_{\vartheta\vartheta}+\tilde{\varphi}_{\zeta\zeta}=\Upsilon(\varphi)\longrightarrow (9)$$

Note that the potential φ_0 on a surface is full. So, the nonlinear component $\tilde{\varphi}$ on a surface is equal to zero. Eq. (9) is solved with the boundary conditions:

$$\begin{matrix}\zeta=0:\tilde{\varphi}=0\\ \zeta\to-\infty:\tilde{\varphi}_\zeta\to 0\end{matrix}\longrightarrow (10)$$

Hence, on a surface $\zeta=0$ an equation (9) takes the form:

$$\tilde{\varphi}_{\zeta\zeta}=\Upsilon(\tilde{\varphi})+\Upsilon(\bar{\varphi})\longrightarrow (11)$$

The derivatives of the linear component $\bar{\varphi}$ in (11) are calculated analytically. Since $\tilde{\varphi}(0)=0$, the equation (11) for velocity potential at $\zeta=0$ takes the form:

$$(1+s)\tilde{w}_\zeta=2\left(\eta_\xi\tilde{w}_\xi+\eta_\vartheta\tilde{w}_\vartheta\right)+\Delta\eta\tilde{w}+r\longrightarrow (12)$$

Here $\tilde{w}=\tilde{\varphi}_\zeta$, $\tilde{w}_\zeta=\tilde{\varphi}_{\zeta\zeta}$, $\bar{w}=\bar{\varphi}_\zeta$, $\bar{w}_\zeta=\bar{\varphi}_{\zeta\zeta}$ and the symbols $\Delta=(\ )_{\xi\xi}+(\ )_{\vartheta\vartheta}$ and $s=\eta_\xi^2+\eta_\vartheta^2$ are used. The term r depends only on the linear component $\bar{\varphi}$

$$r=2\left(\eta_\xi\bar{w}_\xi+\eta_\vartheta\bar{w}_\vartheta\right)+\Delta\eta\bar{w}-s\bar{w}_\zeta\longrightarrow (13)$$

(13) that is calculated analytically using Fourier presentation. The presence of $\tilde{w}_\zeta$ in (12) makes the system of equations unclosed. Since the relation (13) follows formally from Poisson equation, the agreement between these values characterizes accuracy of the entire solution. The calculations of $\tilde{w}_\zeta$ on the basis of an equation (4) and the calculations by surface condition (13) showed a very good agreement shown in Figure 1; [8].

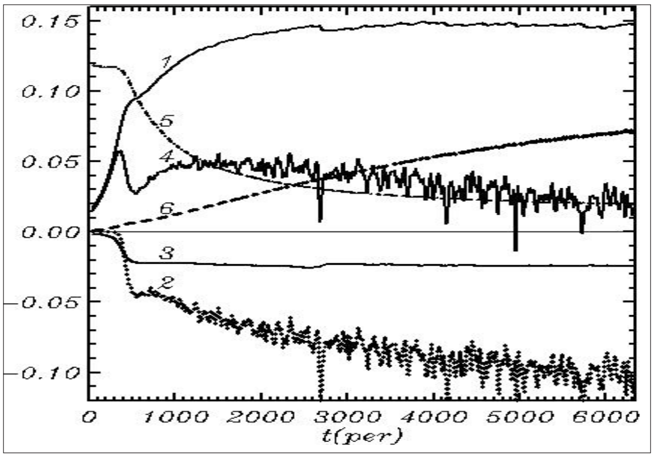

Figure 1: Evolution of integral characteristics of simulate developing wave field: curves 1-4 are the rates of energy transitions, multiplied by:

A. 1-Input energy.

B. 2-Dissipation due to breaking.

C. 3-Dissipation in spectral tail.

D. 4-Balance of energy.

E. 5-Evolution of peak frequency weighted by spectrum divided by 1000.

F. 6-Evolution of potential energy multiplied by Thin straight line shows that transformation of energy in adiabatic part of model is equal to zero.

For evaluation of the connection between $\tilde{w}$ and $\tilde{w}_\zeta$ several thousand short-term numerical experiments with FWM were performed. The calculations were performed at large variations of integral parameters. The problem is reduced to finding dependence

$$A=\frac{w}{w_\zeta}=f(\sigma,\sigma_L,s_t...)\longrightarrow (14)$$

Where $\sigma=\left(\overline{(\eta-\bar{\eta})^2}\right)^{1/2}$ is the dispersion of elevation and $\sigma_L=\left(\overline{(\Lambda-\bar{\Lambda})^2}\right)^{1/2}$ is the dispersion of horizontal Laplacian $\Lambda=\Delta\eta$ while s_t is the averaged steepness of elevation? Finally, the best results were obtained in a form

$$A = \sigma F(\mu) \longrightarrow (15)$$

Where μ is a parameter

$$\mu = \sigma\sigma_L \longrightarrow (16)$$

and function F is approximated by the formula

$$F = \frac{d_0\mu + d_1}{\mu + d_2} \longrightarrow (17)$$

Where $d_0 = 0.535, d_1 = 0.0414, \; d_2 = 0.00321$

The two-dimensional model which is presumably able to replace the full 3-D model (2-5) includes the following three surface equations:

$$\eta_\tau = -\eta_\xi\varphi_\xi - \eta_\vartheta\varphi_\vartheta + (1+s)w \longrightarrow (18)$$

$$\varphi_\tau = -\frac{1}{2}\left(\varphi_\xi^2 + \varphi_\vartheta^2 - (1+s)w^2\right) - \eta - p \longrightarrow (19)$$

$$\tilde{w} = A(1+s)^{-1}\left(2\left(\eta_\xi w_\xi + \eta_\vartheta w_\vartheta\right) + \Delta\eta w - s\bar{w}_\varsigma\right) \longrightarrow (20)$$

Note that the right side of Eq. (20) contains full vertical velocity $w = \bar{w} + \tilde{w}$ as well as the linear component of the vertical velocity $\bar{w}_\varsigma$. The equation is represented in a form convenient for iterations. The iterations were being performed until the following condition was reached.

$$\max\left|\tilde{w}^i - R^{i-1}\right| < 10^{-7} \longrightarrow (21)$$

Where R is the right-hand side of (20) while i is the number of iterations?

The comparison of function F calculated in the course of simulation by FWM with the results of the calculations by formula (17) showed that the agreement between those functions was quite satisfactory. A correlation coefficient between the values F calculated with FWM and those calculated by Eq. (17) is 0.989. The root mean square error is 0.014, which is by two orders smaller than the typical value of F.

The main idea of the current approach is to replace the cumbersome bloc for calculation of the vertical velocity equation for $\tilde{\varphi}$ by a simple 2-D equation (20). Such trick reduces the total 3-D problem to a 2-D one which spares us the trouble of countless problems associated with the numerical solution of Poisson equations. The question is how well these simplified equations reproduce wave dynamics. The most obvious way of validation of the simplified model is performing runs with the identical setting and same initial conditions.

The data on evolution of the integral characteristics for both models were represented in [8]; (Figure 2) which confirmed that HFM could reproduce the integral characteristics quite similar to the results obtained with the full model. The similarity of the results was also illustrated by comparison of the spectra for different statistical characteristics of solution.

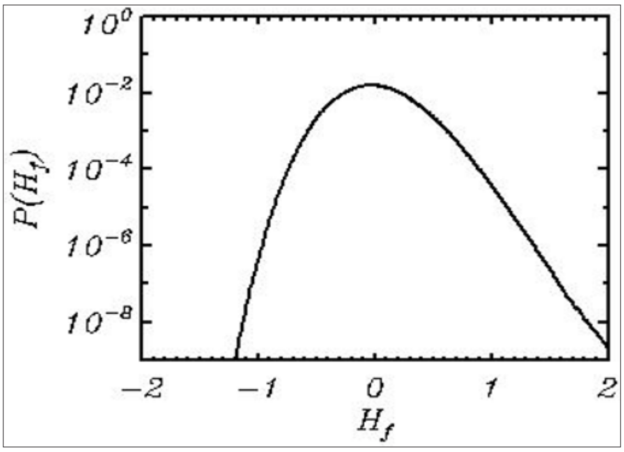

Figure 2: Probability of elevation η normalized by significant wave height H_s.

An Example 0f High-Resolution Simulation of Wave Field Development on The Basis of HFM

Further calculations were carried out with HWM model with initiated algorithms for the input and dissipation of energy. The algorithms for calculation of those terms are described in detail in [1-7]. The last minor modification of the algorithm for breaking was done in [7]. Those algorithms are not described in the current paper.

The model used 257 × 257 Fourier modes, or 1024 × 512 knots. The number of degrees of freedom (the minimum number of Fourier coefficients for elevation η and the surface velocity potential φ is 264,196. In the initial conditions JONSWAP spectrum (Hasselmann et al, 1973) with the peak wave number $k_p = 100$ and an angle distribution proportional to $\left(\mathrm{sch}(\psi)\right)^{256}$ was assigned, which corresponds to nearly unidirected waves with a very small energy. The model was integrated with the time step equal to 0.01 over the period $t = 3000$ As seen, all the characteristics undergo fast transformation during the first period of length $t = 2000$ After that the total input of energy (curve 4) starts to decrease and the evolution of energy approaches an equilibrium level but does not reach it by the end. The frequency of wave peak ω_p (curve 5) decreases as $t^{-1/2}$ which corresponds to the dependence on fetch $\omega_p f^{-1/3}$ which follows from JONSWAP approximation.

An important characteristic is the probability distribution for elevation (Figure 2), which is in good agreement with the results obtained earlier with FWM. For calculations $3.1 \cdot 10^8$ points were used. Note that the results represented in Figure 2 are quite similar to those with the probability results obtained earlier. If we refer freak waves to those with the height exceeding $1.3H_s$ the total probability of such waves is higher than 10^{-6}. As seen in Figure 2, wave height can exceed the value $H_s = 2$ with the probability equal to 10^9. It was not noticed before because no one had ever used such a big volume of data. Obviously, the dependence of probability on wave height is maintained for larger wave height. This again confirms that big waves in a statistically uniform wave field are not generated by some special mechanism, but their physics does not differ (in anything other than size) from the physics of smaller waves.

Conclusion

The paper suggests a new approach to the phase-resolving modeling of two-dimensional periodic wave fields. The only, though an important, shortcoming of the adiabatic 3-D model is quite a low performance connected with solution of 3-D Poisson equation at each time step with iterations over the right side. The runs of such models, even with a medium resolution, take many days or months (depending on the speed of a computer).

In our opinion, the most important result of the current investigation is introducing an exact surface condition (11) connecting the first and second vertical derivatives of the velocity potential. This equation might be useless, however, there are some considerations indicating that these variables could be well connected. The analysis of many thousands of the velocity potential profiles generated by 3-D FWM model proves that this hypothesis does work [8]; (Figure 1). The formulas (15-17) obtained can be inserted into the new surface condition (11) that gives the third surface equation (20) and closes the problem (Eqs. (18-20)). The structure of governing equations for both models is the same; hence, the HWM can be considered as a perturbed FWM. The figures demonstrated in the paper prove that the results generated by the models are at least quite satisfactory.

An obvious advantage of the new scheme is a dramatic increase of the simulation speed. The HWM runs 50-100 times faster than FWM. This ratio can depend on parameterization algorithms as well as on the content and frequency of the results recorded and on the setting of models. At any rate, this ratio remains high. It is obvious that the approach developed here is simplified. It cannot be applied to the individual cases with a small number of modes, for example, for simulation of the steep Stokes wave. The peculiarity of the HWM is that it works well for simulation of the statistically uniform in space wave field with a very large number of modes. It is easy to transform the scheme into a finite-difference form and use the model for investigation of the wave regime in a specific geographical environment. It is important that the model, opposite to FWM, does not contain any global operations, which makes it convenient for parallel computers.

HWM, due to its high performance, can be used as a component of the wave prediction model (like WAM or WAVEWATCH) for interpretation of spectral information in terms of the phase-resolved wave field. The spectra calculated in the spectral model in the selected areas are used for generation of the initial conditions for HWM; then the calculations can be performed up to the stationary statistical regime.

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