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Mini Review

A Brief Review of Publications on the Differential Geometry of Statistical Manifolds

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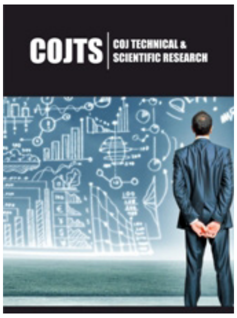

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We present here a brief review of papers and monographs on the differential geometry of statistical manifolds. We do not pretend to the exhaustive completeness of our review. The review is related to the publication of another monograph Ay N, Jost J, Le H. V, Schwachhofer L, Information Geometry, Vol. 64 of a Series of Modern Surveys in Mathematics, Springer Int. Publ. AG, 2017. We recall that Information geometry studies invariant properties of a family of probability distributions and can be applied to various problems in science. Statisticians use statistical models to derive inferences; they use families of probability distributions which form, in most cases, a finite dimensional manifold which in information geometry is called a statistical manifold. Many authors contributed to the development of information geometry or, in other words, geometrical theory of statistics.

The first who founded this theory was Fisher [1]. He has introduced in 1925 the information tensor as an information characterization of a statistical model. The second was Rao [2] in the 1945 paper, he has pointed out that the Fisher information tensor determines a Riemannian metric which is now called the information metric or Fisher metric for the manifold obtained from the family of probability density functions. The third was Efron [3], he exposed in the relationship between statistical curvature and the characteristics of inference. The fifth was Chentsov [4]. He proved in that the Fisher metric is a unique invariant metric. In addition, he has defined a one-parameter group of invariant affine connections in the space of statistic distributions which is now called the Chentsov-Amari connections [5]. Anyone can read more about connections in Chentsov [4] and Amari [6]. After that Amari & Nagaok [6] introduced a conjugate structure or duality structures in information geometry, a finding that has played a fundamental role in development of more applications of information geometry.

In particular, the notion of dually flat metrics was first introduced by Amari & Nagaoka [6] when they studied information geometry on Riemannian spaces. Later, Shen [7] extended the notion of locally dually flatness for Finsler metrics [7]. He identified and studied the dually flat Finsler metrics that are a special and valuable class of Finsler metrics in Finsler geometry, which play a very important role in studying flat Finsler information structures. In addition, Mikesh & Stepanova [8] and Opozda [9] was extend the Bochner technique to information geometry on Riemannian spaces. Moreover, they few global and local theorems on the geometry of statistical structures are proved, for instance, theorems saying that under some topological and geometrical conditions a statistical structure on a Riemannian manifold must be trivial.

We want to highlight how many researchers have contributed to the development of information geometry, its theory and applications. The results of their research are reflected in numerous papers and the following monographs [10-18] etc. Moreover, a number of international work-shops and symposiums on this subject were held: in UK (London, July 10-14, 2000), Italy (Pescara, Sept. 1-5, 2002), Denmark (Leipzig, August 27, 2003), Canada (Toronto, May 8-18, 2004), USA (Ann Arbor, July 29, 2004), Japan (Tokyo, December 12-16, 2005 and Nara, March 6-10, 2012), China (Chenghu, September 3, 2006), France (Paris Saclay Campus, October, 2015) and other countries. The monograph under our review is the eleventh in our list of monographs on Information geometry. Then what distinguishes this monograph from many others? We answer this question as follows. In this monograph the differential geometric approach supplemented by functional analytic considerations. One of the main purposes of this book is to provide a general framework that integrates the differential geometry into the functional analysis.

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