

# Mappings on Fuzzy Soft Topological Spaces

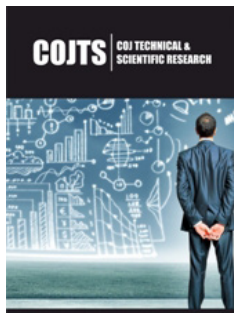
Jingshui Ping\*

School of Finance and Mathematics, China

## Abstract

This paper had been popularized the operational property of fuzzy soft mapping on the basis of fuzzy soft mapping. We had set up a kind of soft mapping and fuzzy soft continuous on fuzzy soft topology and discussed the application of fuzzy soft mapping and its composite properties on the construction of fuzzy soft topology.

**Keywords:** Fuzzy soft set; Fuzzy soft mapping; Fuzzy soft topology; Fuzzy soft continuous



\*Corresponding author: Jingshui Ping, School of Finance and mathematics, Huainan, China

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## Introduction

Molodtsov put forward the soft set theory in order to handling uncertain problems in 1999. This theory had overcome a main shortcoming of mathematical methods; that was lacking parameter tool. The soft set theory has been successfully applied in several fields such as decision analysis, pattern recognition and data collection. Maji [1] perfected the theoretical system of soft set and studied the fuzzy soft set theory. Paper included the research of the algebraic structure of soft set; and Hazra [2] had set up the soft topology construction. In Kharal & Ahmad [3] began to study fuzzy soft mapping. In 2011, Bekir T & Burc K [4] set up a fuzzy soft topology structure. In 2012, Abdülkadir [5] studied some properties of soft mapping on soft topology. Based on all these mentioned above, we research on fuzzy soft mapping on fuzzy soft topology and its application in fuzzy soft topology structure [6].

## Problem Statement and Preliminaries

### Definition 1

[7] Let  $A \subset E$  and  $F(U)$  be the set of all fuzzy sets in  $U$ . Then the pair  $(f, A)$  is called a fuzzy soft set over  $U$ , where  $f : A \rightarrow F(U)$  is a function. A fuzzy soft set  $(f, A)$  can be viewed [2]  $(f, A) = \{a = \{u_{f_a(u)} | u \in U\} \}$  [8-12].

### Definition 2

[13] Let  $\tilde{g} = (h, p)$ , where  $h : U \rightarrow U'$  and  $p : E \rightarrow E'$  be mappings. Let  $(f, A)$  and  $(k, C)$  be fuzzy soft sets, over  $U$  and  $U'$ , where  $A \subset E$  and  $C \subset E'$ ,

A fuzzy soft image of a fuzzy soft set  $(f, A)$  under mapping  $\tilde{g}$  is defined as follows:

$$\tilde{g}(f, A) = \{a' = \{u'_{f'_a(u')} | u' \in U', a' \in E'\} \text{ where}$$

$$f'_a(u') = \begin{cases} \bigvee_{u \in h^{-1}(u')} \bigvee_{a \in p^{-1}(a') \cap A} f_a(u), & \text{if } h^{-1}(u') \neq \emptyset \text{ and } p^{-1}(a') \neq \emptyset \\ 0, & \text{otherwise} \end{cases};$$

A fuzzy soft inverse image of a fuzzy soft set  $(k, C)$  under mapping  $\tilde{g}$  is defined as follows:  $\tilde{g}^{-1}(k, C) = \{a = \{u_{t_a(u)} | u \in U, a \in E\}$ , where

$$t_a(u) = \begin{cases} k_{p(a)}(h(u)), & \text{if } p(a) \in C \\ 0, & \text{if } p(a) \notin C \end{cases}$$

**Remark 1:** The definition mentioned above, actually, is a kind of improvement of 3.2 definition in document [13].

Let  $\tilde{g} = (h, p)$  where  $h : U \rightarrow U'$  and  $p : E \rightarrow E'$  be mappings.

Let  $(f_i, A_i)$  and  $(g_j, B_j)$  be fuzzy soft sets over  $U$  and  $U'$  respectively, where

$A_i \subset E, B_i \subset E', i = 1, 2$  According to [13], we have

**Theorem 1:**

- (1)  $\tilde{g}(\tilde{\Phi}_{A_i}) = \tilde{\Phi}_{E'}, \tilde{g}^{-1}(\tilde{\Phi}_{B_i}) = \tilde{\Phi}_E$ ;
  - (2)  $\tilde{g}[f_1, A_1] \tilde{U}(f_2, A_2) = \tilde{g}(f_1, A_1) \tilde{U} \tilde{g}(f_2, A_2)$ ;
  - (3)  $\tilde{g}[f_1, A_1] \tilde{\cap}(f_2, A_2) = \tilde{g}(f_1, A_1) \tilde{\cap} \tilde{g}(f_2, A_2)$ ;
  - (4) if  $(f_1, A_1) \tilde{\subset} (f_2, A_2), (g_1, B_1) \tilde{\subset} (g_2, B_2)$ ;
- then  $\tilde{g}(f_1, A_1) \tilde{\subset} \tilde{g}(f_2, A_2)$ ,
- $\tilde{g}^{-1}(g_1, B_1) \tilde{\subset} \tilde{g}^{-1}(g_2, B_2)$ ;
- (5)  $\tilde{g}^{-1}[f_1, A_1] \tilde{U}(f_2, A_2) = \tilde{g}^{-1}(f_1, A_1) \tilde{U} \tilde{g}^{-1}(f_2, A_2)$ ;
  - (6)  $\tilde{g}^{-1}[f_1, A_1] \tilde{\cap}(f_2, A_2) = \tilde{g}^{-1}(f_1, A_1) \tilde{\cap} \tilde{g}^{-1}(f_2, A_2)$ .

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**Definition 3**

[14] Let  $(\gamma, X)$  be an element of  $F(U; E), P(\gamma, X)$  be the set of all fuzzy soft subsets of  $(\gamma, X)$  and  $\tilde{\tau}$  be a subfamily of  $P(\gamma, X)$ . Then  $\tilde{\tau}$  is called fuzzy soft topology on  $(\gamma, X)$  if the following conditions are satisfied:

- $\tilde{\Phi}_{X, (\gamma, X)} \in \tilde{\tau}$ ;
- $(f, A) (g, B) \in \tilde{\tau} \Rightarrow (f, A) \tilde{\cap} (g, B) \in \tilde{\tau}$ ;
- $\{f_k, A_k | k \in K\} \subset \tilde{\tau} \Rightarrow \tilde{U}_{k \in K} (f_k, A_k) \in \tilde{\tau}$ .

The pair  $(X_\gamma, \tilde{\tau})$  is called a fuzzy soft topological space.

Let  $(X_\gamma, \tilde{\tau})$  be a fuzzy soft topological space and  $(f, A) \in P(\gamma, X)$ . Then the collection

$\tilde{\tau}_{(f, A)} = \{f, A) \tilde{\cap} (g, B) | (g, B) \in \tilde{\tau}\}$  is a fuzzy soft topology on the fuzzy soft subset  $(f, A)$  relative to parameter set  $A$ .

**Theorem 2:** Let  $\tilde{g} = (h, p)$ , where  $h : U \rightarrow U'$  and  $p : E \rightarrow E'$  be mappings. Let  $(k, C)$  be fuzzy soft set, over  $U'$ , where  $C \subset E'$ . Then we have  $\tilde{g}^{-1}(k, C) = \tilde{g}^{-1}(k, E')$  [15].

Proof: According to definition 2, we have

$$\tilde{g}^{-1}(k, C) = \{a = \{u_{t_a(u)} | u \in U, a \in E'\} ,$$

$$\text{where } t_a(u) = \begin{cases} k_{p(a)}(h(u)), & \text{if } p(a) \in C, \text{ and} \\ 0 & , \text{ if } p(a) \notin C \end{cases}$$

$$\tilde{g}^{-1}(k, E') = \{a = \{u_{s_a(u)} | u \in U, a \in E'\}$$

$$\text{where } s_a(u) = k_{p(a)}(h(u)) =$$

$$\begin{cases} k_{p(a)}(h(u)), & \text{if } p(a) \in C \\ 0, & \text{if } p(a) \notin C \end{cases} = t_a(u).$$

$$\text{Hence } \tilde{g}^{-1}(k, C) = \tilde{g}^{-1}(k, E').$$

**Theorem 3:** Let  $\tilde{g} = (h, p)$  where  $h : U \rightarrow U'$  and  $p : E \rightarrow E'$  be mappings.

Let  $(f, A)$  and  $(k, E')$  be fuzzy soft sets over  $U$  and  $U'$  respectively, where

$A \subset E$ , we have

$$(f, A) \tilde{\subset} \tilde{g}^{-1}(\tilde{g}(f, A)) ;$$

$$\tilde{g}(\tilde{g}^{-1}(k, E')) \tilde{\subset} (k, E').$$

Proof (1) Let  $(f, A) = \{a = \{u_{f_a(u)} | u \in U\}$ .

Then  $\tilde{g}(f, A) = (f', E') = \{a' = \{u'_{f'_a(u')} | u' \in U'\}$

where

$$f'_a(u') = \begin{cases} \bigvee_{a \in p^{-1}(a') \cap A} \bigvee_{u \in h^{-1}(u')} f_a(u), & p^{-1}(a') \cap A \neq \emptyset, h^{-1}(u') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{g}^{-1}(f', E') = (f'', E) = \{a = \{u_{f''_a(u)} | u \in U\}$$

Since  $p(a) \subset E'$ , then  $f''_a(u) = f'_{p(a)}(h(u))$

$$= \bigvee_{a \in p^{-1}(p(a)) \cap A} \bigvee_{u \in h^{-1}(h(u))} f_a(u).$$

For  $p^{-1}(p(a)) \cap A = A, h^{-1}(h(u)) = U$ , we have

$$f''_a(u) \geq f_a(u).$$

Hence  $(f, A) \tilde{\subset} \tilde{g}^{-1}(\tilde{g}(f, A))$ .

(2) Let  $(k, E') = \{a' = \{u'_{k'_a(u')} | u' \in U'\}$ .

Then  $\tilde{g}^{-1}(k, E') = (k', E) = \{a = \{u_{k'_a(u)} | u \in U\}$ , where  $k'_a(u) = k'_{p(a)}(h(u))$

$$\tilde{g}(k', E) = (k'', E') = \{a' = \{u'_{k''_a(u')} | u' \in U'\}$$

Since  $k''_a(u') \leq \bigvee_{a \in p^{-1}(a') \cap E} \bigvee_{u \in h^{-1}(u')} k'_a(u)$

$$= \bigvee_{a \in p^{-1}(a') \cap E} \bigvee_{u \in h^{-1}(u')} k_{p(a)}(h(u))$$

$$p(p^{-1}(a')) = a', h(h^{-1}(u')) = u',$$

Then we have  $k''_a(u') \leq k'_a(u')$ . Hence

$$\tilde{g}(\tilde{g}^{-1}(k, E')) \tilde{\subset} (k, E').$$

**Theorem 4:** Let  $\tilde{g} = (h, p)$ , where  $h : U \rightarrow U'$  and  $p : E \rightarrow E'$  be mappings.

Let  $(f, A)$  and  $(k, B)$  be fuzzy soft sets, over  $U$  and  $U'$ , where  $A \subset E$  and

$C \subset E'$ . Then we have

$$g(\tilde{g}^{-1}(\tilde{g}(f, A))) = \tilde{g}(f, A);$$

$$\tilde{g}^{-1}(\tilde{g}(\tilde{g}^{-1}(k, B))) = \tilde{g}^{-1}(k, B).$$

**Proof 1:** According to Theorem 3,  $(f, A) \tilde{\subset} \tilde{g}^{-1}(\tilde{g}(f, A))$ ,  $\tilde{g}(\tilde{g}^{-1}(k, E')) \tilde{\subset} (k, E')$

Then we have  $\tilde{g}(f, A) \tilde{\subset} \tilde{g}(\tilde{g}^{-1}(\tilde{g}(f, A))) \tilde{\subset} \tilde{g}(f, A)$ .

Hence  $g(\tilde{g}^{-1}(\tilde{g}(f, A))) = \tilde{g}(f, A)$ ;

(2) According to (1)  $\tilde{g}(f, A) = g(\tilde{g}^{-1}(\tilde{g}(f, A)))$ , let  $\tilde{g}(f, A) = (k, E')$ , then

$$(k, E') = g(\tilde{g}^{-1}(k, E')). \text{ Therefore}$$

$$\tilde{g}^{-1}(k, E') = \tilde{g}^{-1}(\tilde{g}(\tilde{g}^{-1}(k, E'))).$$

According to Theorem 2,  $\tilde{g}^{-1}(k, E') = \tilde{g}^{-1}(k, B)$ . Hence

$$\tilde{g}^{-1}(\tilde{g}(\tilde{g}^{-1}(k, B))) = \tilde{g}^{-1}(k, B).$$

**Definition 4**

Let  $(X, \tilde{\tau})$  and  $(X', \tilde{\tau}')$  be fuzzy soft topological spaces over  $U$  and  $U'$ , respectively. Let  $\tilde{g} = (h, p)$  where  $h: U \rightarrow U'$  and  $p: X \rightarrow X'$  be mappings.  $\tilde{g}$  is said to be fuzzy soft continuous if for each  $(k, C) \in \tilde{\tau}, \tilde{g}^{-1}(k, C) \in \tilde{\tau}$ .

**Theorem 5:** Let  $(X, \tilde{\tau})$  and  $(X', \tilde{\tau}')$  be fuzzy soft topological spaces over  $U$  and  $U'$ , respectively. Let  $\tilde{g} = (h, p)$  where  $h: U \rightarrow U'$  and  $p: X \rightarrow X'$  be mappings.

We have,

$\tilde{g}^{-1}(\tilde{\tau}') = \{\tilde{g}^{-1}(k, C) | (k, C) \in \tilde{\tau}'\}$  is a fuzzy soft topology on  $\tilde{g}^{-1}(\gamma', X')$ ;

$\tilde{g}^{-1}(\tilde{\tau}'_{(k,C)}) = \{\tilde{g}^{-1}[k, C] \tilde{\cap} (k', C') | (k', C') \in \tilde{\tau}'\}$  is a fuzzy soft topology on  $\tilde{g}^{-1}(k, C)$ .

Proof (1) For  $\tilde{\Phi}_{X'} \in \tilde{\tau}', (\gamma', X') \in \tilde{\tau}'$ , we have

$$\tilde{g}^{-1}(\tilde{\Phi}_{X'}) = \tilde{\Phi}_X \in \tilde{g}^{-1}(\tilde{\tau}') \text{ and } \tilde{g}^{-1}(\gamma', X') \in \tilde{g}^{-1}(\tilde{\tau}').$$

If  $\tilde{g}^{-1}(g_1, B_1) \in \tilde{g}^{-1}(\tilde{\tau}')$  and  $\tilde{g}^{-1}(g_2, B_2) \in \tilde{g}^{-1}(\tilde{\tau}')$ ,

$$\text{Then } \tilde{g}^{-1}(g_1, B_1) \tilde{\cap} \tilde{g}^{-1}(g_2, B_2) = \tilde{g}^{-1}[g_1, B_1] \tilde{\cap} (g_2, B_2) \in \tilde{g}^{-1}(\tilde{\tau}').$$

Let  $\tilde{g}^{-1}(g_k, B_k) \in \tilde{g}^{-1}(\tilde{\tau}'), k \in K$ ,

$$\text{Then } \tilde{\cup}_{k \in K} \tilde{g}^{-1}(g_k, B_k) = \tilde{g}^{-1}[\tilde{\cup}_{k \in K} (g_k, B_k)] \in \tilde{g}^{-1}(\tilde{\tau}').$$

(2) For  $(k', C') \tilde{\Phi}_{X'} \in \tilde{\tau}'$ , we have  $\tilde{g}^{-1}[k, C] \tilde{\cap} (k', C') \tilde{\subset} \tilde{g}^{-1}(k, C)$  and

$$\tilde{g}^{-1}[k, C] \tilde{\cap} \tilde{\Phi}_{X'} = \tilde{g}^{-1}(k, C) \tilde{\cap} \tilde{\Phi}_{X'} \\ \tilde{g}^{-1}(\tilde{\Phi}_{X'}) = (s, X) \tilde{\cap} \tilde{\Phi}_X = \tilde{\Phi}_X \in \tilde{g}^{-1}(\tilde{\tau}'_{(k,C)}),$$

For  $(k, C) \tilde{\subset} (\gamma', X'), \tilde{g}^{-1}(k, C) = \tilde{g}^{-1}[k, C] \tilde{\cap} (\gamma', X') \in \tilde{g}^{-1}(\tilde{\tau}'_{(k,C)})$ .

Suppose that  $\tilde{g}^{-1}[k, C] \tilde{\cap} (g_1, B_1), \tilde{g}^{-1}[k, C] \tilde{\cap} (g_2, B_2) \in \tilde{g}^{-1}(\tilde{\tau}'_{(k,C)})$ . Then

$$\tilde{g}^{-1}[k, C] \tilde{\cap} (g_1, B_1) \tilde{\cap} \tilde{g}^{-1}[k, C] \tilde{\cap} (g_2, B_2) = \\ \tilde{g}^{-1}[k, C] \tilde{\cap} (g_1, B_1) \tilde{\cap} (g_2, B_2) \in \tilde{g}^{-1}(\tilde{\tau}'_{(k,C)})$$

Let  $\tilde{g}^{-1}[k, C] \tilde{\cap} (g_k, B_k) \in \tilde{g}^{-1}(\tilde{\tau}'_{(k,C)}), k \in K$ .

$$\text{Then } \tilde{\cup}_{k \in K} \tilde{g}^{-1}[k, C] \tilde{\cap} (g_k, B_k) = \\ \tilde{g}^{-1}[k, C] \tilde{\cap} (\tilde{\cup}_{k \in K} (g_k, B_k)) \in \tilde{g}^{-1}(\tilde{\tau}'_{(k,C)}).$$

**Definition 5**

Let  $h: U \rightarrow U', k: U' \rightarrow U'', p: E \rightarrow E', q: E' \rightarrow E''$  be mappings.

Let  $\tilde{f} = (h, p)$   $\tilde{g} = (k, q)$ . The composite  $\tilde{g} \circ \tilde{f}$  of the mappings  $\tilde{f}$  and  $\tilde{g}$  defined

by  $\tilde{g} \circ \tilde{f} = (k \circ h, q \circ p)$ , where  $k \circ h$  and  $q \circ p$  are composites of mappings  $h$  and  $k, p$  and  $q$ , respectively.

**Theorem 6:** Let  $(m, A)$   $(n, B)$  be fuzzy soft sets over  $U$  and  $U'$ , respectively. Then we have

$$(1) \tilde{g}(\tilde{f}(m, A)) = (\tilde{g} \circ \tilde{f})(m, A);$$

$$(2) (\tilde{g} \circ \tilde{f})^{-1}(n, B) = (\tilde{f}^{-1} \circ \tilde{g}^{-1})(n, B) = \tilde{f}^{-1}(\tilde{g}^{-1}(n, B)).$$

Proof (1) Let

$$\tilde{f}(m, A) = (m', E') = \{a' = \{u'_{m'_a(u')} | u' \in U'\} \}, \text{ where}$$

$$m'_a(u') = \begin{cases} \bigvee_{a' \in p^{-1}(a') \cap A} \bigvee_{u' \in h^{-1}(u')} m_a(u), & p^{-1}(a') \cap A \neq \emptyset, h^{-1}(u') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{g}(m', E') = (m'', E'') = \{a'' = \{u''_{m''_a(u'')} | u'' \in U''\} \}$$

where

$$m''_a(u'') = \begin{cases} \bigvee_{a' \in q^{-1}(a'') \cap E'} \bigvee_{u' \in k^{-1}(u'')} m'_a(u'), & q^{-1}(a'') \cap E' \neq \emptyset, k^{-1}(u'') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{a' \in q^{-1}(a'') \cap E'} \bigvee_{u' \in k^{-1}(u'')} m'_a(u'), & q^{-1}(a'') \neq \emptyset, k^{-1}(u'') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{a' \in q^{-1}(a'') \cap E'} \bigvee_{u' \in k^{-1}(u'')} \bigvee_{a \in p^{-1}(a') \cap A} \bigvee_{u' \in h^{-1}(u')} m_a(u), & q^{-1}(a'') \neq \emptyset, k^{-1}(u'') \neq \emptyset, p^{-1}(a') \cap A \neq \emptyset, h^{-1}(u') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{a' \in p^{-1}(q^{-1}(a'')) \cap A} \bigvee_{u' \in h^{-1}(k^{-1}(u''))} m_a(u), & p^{-1}(q^{-1}(a'')) \cap A \neq \emptyset, h^{-1}(k^{-1}(u'')) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{a' \in p^{-1}(q^{-1}(a'')) \cap A} \bigvee_{u' \in h^{-1}(k^{-1}(u''))} m_a(u), & (q \circ p)^{-1}(a'') \cap A \neq \emptyset, (k \circ h)^{-1}(u'') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

Let

$$(\tilde{g} \circ \tilde{f})(m, A) = (t, E'') = \{a'' = \{u''_{t_a(u'')} | u'' \in U''\} \} \text{ then}$$

$$t_a(u'') = \begin{cases} \bigvee_{a' \in p^{-1}(q^{-1}(a'')) \cap A} \bigvee_{u' \in h^{-1}(k^{-1}(u''))} m_a(u), & (q \circ p)^{-1}(a'') \cap A \neq \emptyset, (k \circ h)^{-1}(u'') \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$= m''_a(u'')$$

(2) Let  $(\tilde{g} \circ \tilde{f})^{-1}(n, B) = (s, E) = \{a = \{u_{s_a(u)} | u \in U\} \}$ , where

$$s_a(u) = \begin{cases} n_{(q \circ p)(a)}(k \circ h(u)), & (q \circ p)(a) \in B \\ 0, & (q \circ p)(a) \notin B \end{cases}$$

Let  $\tilde{g}^{-1}(n, B) = (n', E') = \{a' = \{u'_{n'_a(u')} | u' \in U'\} \}$ , where

$$n'_a(u') = \begin{cases} n_{q(a)}(k(u')), & q(a) \in B \\ 0, & q(a) \notin B \end{cases}$$

Let  $\tilde{f}^{-1}(n', E') = (n'', E) = \{a = \{u_{n''_a(u)} | u \in U\} \}$ , where

$$n''_a(u) = \begin{cases} n'_{p(a)}(h(u)), & p(a) \in E' \\ 0, & p(a) \notin E' \end{cases} = n'_{p(a)}(h(u))$$

$$= \begin{cases} n_{q(p(a))}(k(h(u))), & q(p(a)) \in B \\ 0, & q(p(a)) \notin B \end{cases} = s_a(u)$$

**Theorem 7:** Let  $(X_\gamma, \tilde{\tau})$   $(Y_\gamma, \tilde{\tau}')$  and  $(Z_\gamma, \tilde{\tau}'')$  be fuzzy soft topological spaces over  $U, U'$  and  $U''$ , respectively. Let  $\tilde{g} = (h, p)$  and  $\tilde{g}' = (h', p')$ , where  $h: U \rightarrow U'$ ,  $h': U' \rightarrow U''$  and  $p: X \rightarrow Y$ ,  $p': Y \rightarrow Z$  be mappings.

We have

If  $\tilde{g}$  is fuzzy soft continuous then for each  $(f, A) \tilde{\subset} (\gamma, X)$  and each neighborhood  $(v, V)$  of  $\tilde{g}(f, A)$ , there is a neighborhood  $(k, C)$  of  $(f, A)$  such that  $\tilde{g}(k, C) \tilde{\subset} (v, V)$ .

If  $\tilde{g}$  and  $\tilde{g}'$  are both fuzzy soft continuous then  $\tilde{g} \circ \tilde{g}'$  is fuzzy soft continuous.

Proof (1): For each neighborhood  $(v, V)$  of  $\tilde{g}(f, A)$ , there exists an open fuzzy soft set  $(m, D) \in \tilde{\tau}'$  such that  $\tilde{g}(f, A) \tilde{\subset} (m, D) \tilde{\subset} (v, V)$ , Then  $g^{-1}(\tilde{g}(f, A)) \tilde{\subset} g^{-1}(m, D) \tilde{\subset} g^{-1}(v, V)$ , Since  $\tilde{g}$  is fuzzy soft continuous, we have  $g^{-1}(m, D) \in \tilde{\tau}$ .

For  $(f, A) \tilde{\subset} g^{-1}(\tilde{g}(f, A))$ , we have  $(f, A) \tilde{\subset} g^{-1}(m, D) \tilde{\subset} g^{-1}(v, V)$   $g^{-1}(v, V)$  is a neighborhood of  $(f, A)$ .

Let  $(k, C) = g^{-1}(v, V)$ , then  $\tilde{g}(k, C) = g(\tilde{g}^{-1}(v, V)) \tilde{\subset} (v, V)$ .

(2) If  $(j, F) \in \tilde{\tau}''$  then  $\tilde{g}'^{-1}(j, F) \in \tilde{\tau}'$  and  $\tilde{g}^{-1}(\tilde{g}'^{-1}(j, F)) \in \tilde{\tau}$ .

But  $\tilde{g}^{-1}(\tilde{g}'^{-1}(j, F)) = (\tilde{g}' \circ \tilde{g})^{-1}(j, F)$ , so that  $\tilde{g}' \circ \tilde{g}$  is fuzzy soft continuous.

**Theorem 8:** Let  $\tilde{g} = (h, p)$  where  $h: U \rightarrow U'$  and  $p: X \rightarrow X'$  be mappings. Let  $\tilde{\tau}$  be fuzzy soft topology on  $g^{-1}(k, B)$ . Then  $\tilde{\tau}' = \{ (f, A) \tilde{\subset} (k, B) | g^{-1}(f, A) \in \tilde{\tau} \}$  is a fuzzy soft topology on  $(k, B)$ .

Proof (1): Since  $\tilde{g}^{-1}(k, B) \in \tilde{\tau}$ ,  $\tilde{g}^{-1}(\tilde{\Phi}_B) = \tilde{\Phi}_E \in \tilde{\tau}$ , then  $(k, B) \in \tilde{\tau}'$ ,  $\tilde{\Phi}_B \in \tilde{\tau}'$

(2) Suppose that  $(f_1, A_1) (f_2, A_2) \in \tilde{\tau}'$ . Then  $(f_1, A_1) \tilde{\cap} (f_2, A_2) \tilde{\subset} (k, B)$ ,

$\tilde{g}^{-1}[(f_1, A_1) \tilde{\cap} (f_2, A_2)] = \tilde{g}^{-1}(f_1, A_1) \tilde{\cap} \tilde{g}^{-1}(f_2, A_2) \in \tilde{\tau}$ , we have

$(f_1, A_1) \tilde{\cap} (f_2, A_2) \in \tilde{\tau}'$ .

(3) Let  $\{ (f_k, A_k) | k \in K \}$  be a subfamily of  $\tilde{\tau}'$ . Then  $\bigcup_{k \in K} (f_k, A_k) \tilde{\subset} (k, B)$ ,

$\tilde{g}^{-1}[\bigcup_{k \in K} (f_k, A_k)] = \bigcup_{k \in K} \tilde{g}^{-1}(f_k, A_k) \in \tilde{\tau}$ , so that

$\bigcup_{k \in K} (f_k, A_k) \in \tilde{\tau}'$ .

## Conclusion

Fuzzy soft mapping is an extremely significant part in studying the spacious property of fuzzy soft topology. This paper popularizes fuzzy soft mapping as well as the operational properties of its inverse and composite operation. We set up the fuzzy soft continuous on fuzzy soft topology and study the application of fuzzy soft mapping's various properties in the construction of fuzzy soft topology. As for the question that how to establish fuzzy soft quotient topology space by fuzzy soft mapping, detailed information will be given in author's another paper.

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