Crimson Publishers
Wings to the Research

Review Article

Fuzzy Sets Extensions and Intelligence

Kahraman C* and Boltürk E

Department of Industrial Engineering, Turkey

Abstract

It is quite hard to model human thoughts because of the lack of representation them by 0-1 logic. Fuzzy logic is a continuous and boundless valued logic whose values are between 0 and 1 and can be modeled by the fuzzy set theory. On the other hand, intelligent techniques such as machine learning, soft computing, neural networks, etc. are widely used in literature in order to better modeling and analyzing complex problems. It can be easily seen in the literature that fuzzy set theory and related extensions are integrated with these intelligent techniques. This paper illustrates a review on fuzzy sets theory and its extensions and intelligent techniques.

Keywords: Fuzzy set theory; Extensions; Intelligence; Intelligent techniques

ISSN: 2832-4463

***Corresponding author:** Kahraman C, Department of Industrial Engineering, Turkey

Submission: March 29, 2021

Published: June 03, 2021

Volume 1 - Issue 4

How to cite this article: Kahraman C, Boltürk E. Fuzzy Sets Extensions and Intelligence. COJ Rob Artificial Intel. 1(4). COJRA. 000519. 2021.
DOI: 10.31031/COJRA.2021.01.000519

Introduction

Intelligent systems and systems which use intelligent computing techniques are able to decide and think as human. These systems have been needed to be developed for solving complex problems where the classical methods cannot find an optimal solution. Robotics, intelligent control knowledge-based paradigms, computational intelligence, learning paradigms, soft computing including neural networks, fuzzy systems, social intelligence, ambient intelligence, computational neuroscience, artificial life, virtual worlds and society, trust management cognitive science and systems, interactive entertainment, human-centered/human-centric computing, intelligent data analysis, knowledge management, intelligent agents, intelligent decision making and support, intelligent network security, web intelligence and multimedia are some of the problem areas that intelligent computing and intelligent systems are employed. The fuzzy sets theory was introduced by Zadeh [1] in 1965. Since ordinary fuzzy sets have been developed, a number of extensions have been proposed by many researchers. The extensions of ordinary fuzzy sets are type 2 fuzzy sets [2], intuitionistic fuzzy sets [3], fuzzy multisets [4], intuitionistic fuzzy sets of second type [5], neutrosophic fuzzy sets [6], nonstationary fuzzy sets [7], hesitant fuzzy sets [8], Pythagorean fuzzy sets [9], picture fuzzy sets [10], q-rung fuzzy sets [11], fermatean fuzzy sets [12], spherical fuzzy sets [13] and circular intuitionistic fuzzy sets [14]. These extensions have been used such as decision making, forecasting, controlling, and engineering economics with intelligent systems. In this study, we try to summarize the papers involving the integrated models of fuzzy sets and intelligent systems. The literature review with graphical illustrations is given in Section 2. Section 3 summarizes the extensions of fuzzy sets. The conclusions are given in Section 3.

Literature Review

The integration of "Fuzzy sets" and "Intelligence" concepts have been studied in the literature since 1972. In this section, we try to summarize "fuzzy sets" and "intelligence" related publications based on the Scopus database. There are 10,671 documents and these are analyzed in (Figures 1-8). Figure 1 shows the pattern of fuzzy sets and intelligence publications with respect to years. It is seen that most of the studies have been published in 2010 with a rate of 6%. It is observed that there has been an increase in number of publications since 1992. In Figure 2, the distribution of Fuzzy sets and intelligence papers by their sources are given. Most of the publications on "Fuzzy Sets" and "Intelligence" papers have been published in Lecture Notes in Computer Science Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics, Fuzzy Sets and Systems and IEEE International Conference on Fuzzy Systems, respectively. In Figure 3, the publication percentages of authors on fuzzy sets and intelligence are shown. There are 159 authors in this area and Pedrycz W, Yager RR, and Mesiar R are the leading authors on fuzzy sets and intelligence, respectively.

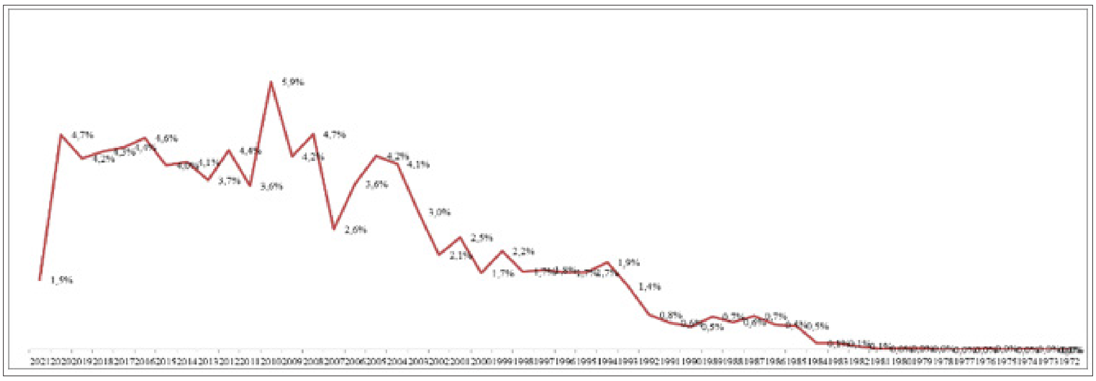

Figure 1: Distribution of fuzzy sets and intelligence papers with respect to years.

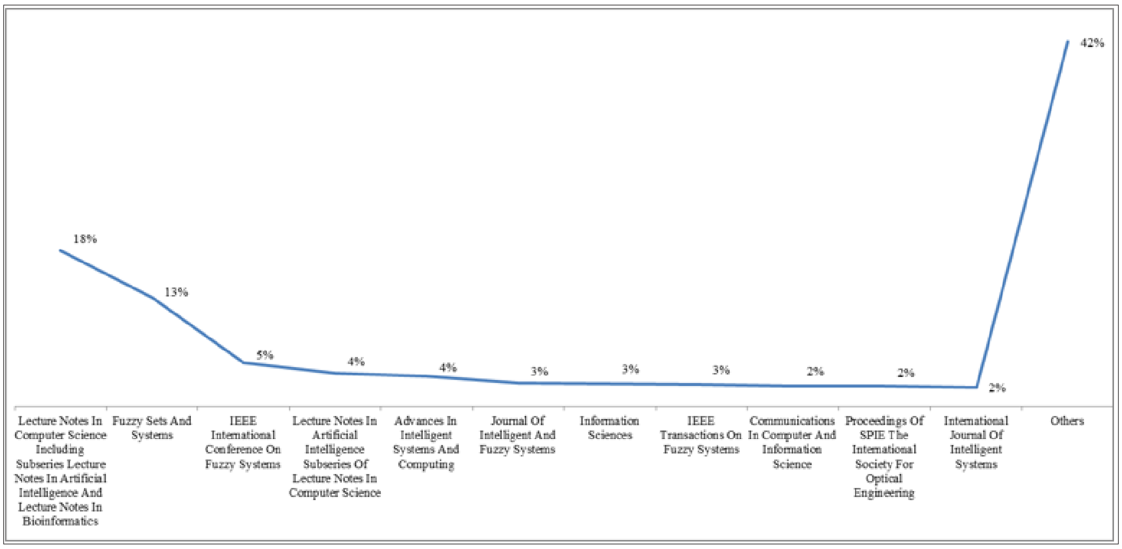

Figure 2: Fuzzy sets and intelligence publications by their published sources.

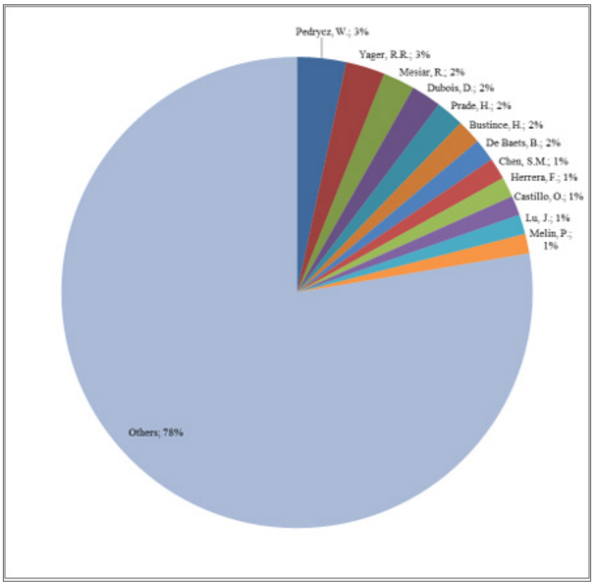

Figure 3: Publication percentages of authors on fuzzy sets and intelligence.

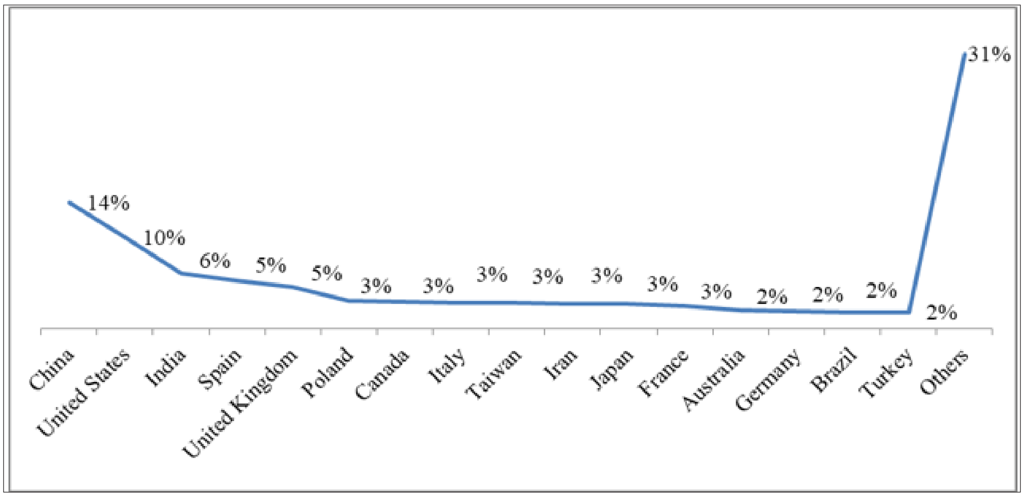

Figure 4: Distribution of fuzzy sets and intelligence publications by their countries.

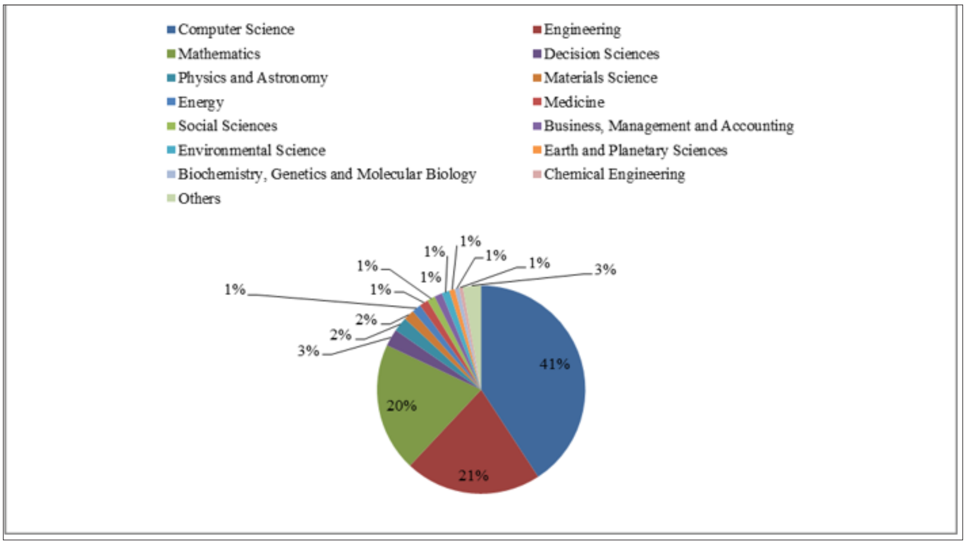

Figure 5: Distribution of fuzzy sets and intelligence papers by their subjects areas.

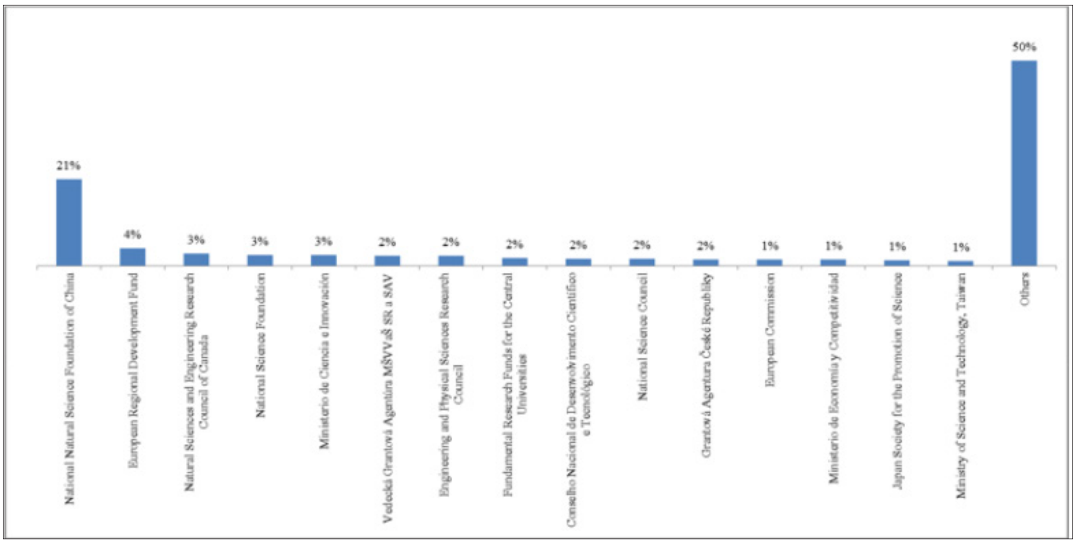

Figure 6: Percentages of publications for funding sponsors on fuzzy sets and intelligence.

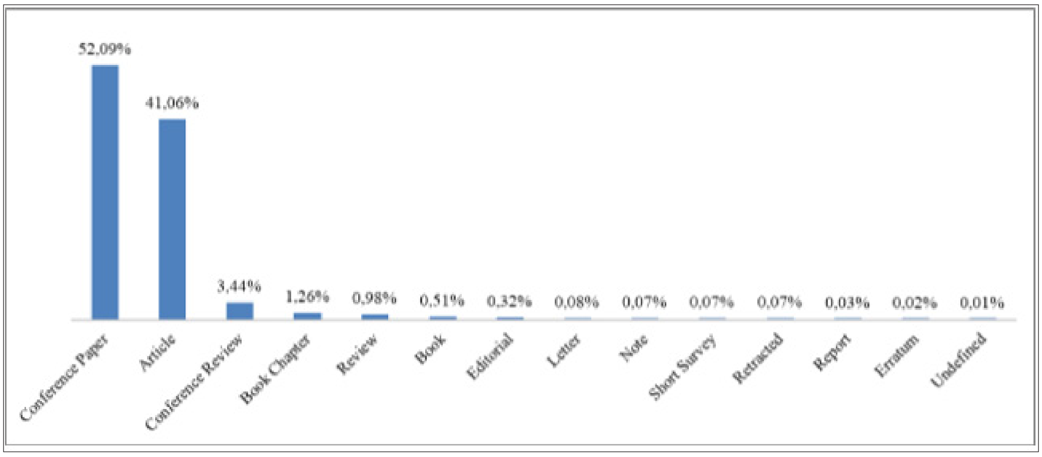

Figure 7: Percentages of Publication types on Fuzzy sets and intelligence.

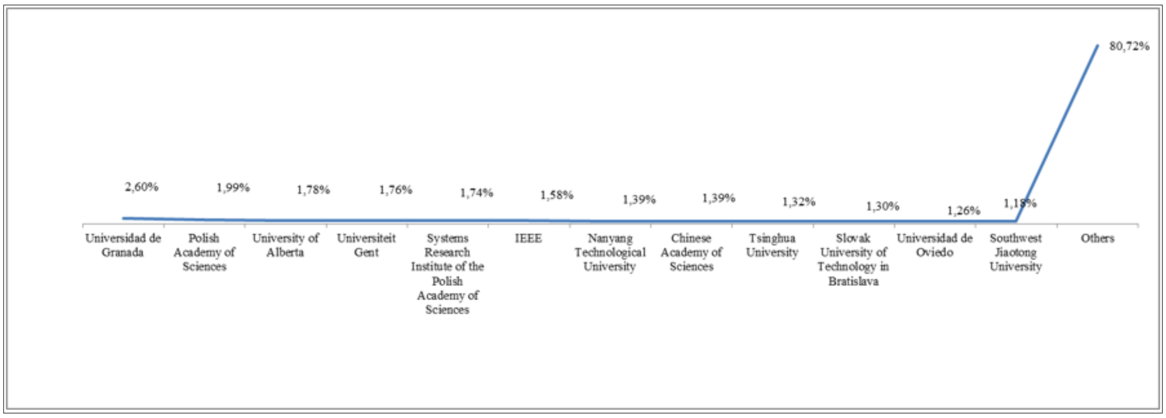

Figure 8: Percentages of affiliations those have fuzzy sets and intelligence publications.

The distribution of fuzzy sets and intelligence publications with respect to their source countries is illustrated in Figure 4. China, United States and India are the first three leaders among others.

The distribution of fuzzy sets and intelligence publications with respect to their subjects is illustrated in Figure 5. Computer science, engineering and mathematics are the first three subject areas among others. The other subjects are neuroscience, agricultural and biological sciences, chemistry, economics, econometrics and finance, arts and humanities, health professions, multidisciplinary, psychology, pharmacology, toxicology and pharmaceutics, immunology and microbiology, nursing, dentistry, and veterinary. The distribution of fuzzy sets and intelligence publications with respect to their funding sponsors is illustrated in Figure 6. National Natural Science Foundation of China is the leader among other funding sponsors. The distribution of document types on Fuzzy Sets and Intelligence are given in Figure 7. Most common types of publications are conference papers with a percentage of 52.09% and articles with a percentage of 41.06%. The distribution of affiliation has fuzzy sets and intelligence publications is illustrated in Figure 8. Universidad de Granada with a percentage of 2.6%, Polish Academy of Sciences with a percentage of 1.99 % and University of Alberta with a percentage of 1.78% are in the first three positions among other affiliations.

Extensions of Fuzzy Sets

A way of describing a fuzzy set is to list ordered pairs: an object X and its membership degree $\mu_A(x)\in[0,1]$ in a set $\tilde{A}$. To describe an ordinary fuzzy set, the following notation proposed by Zadeh (1965) can be used:

$$\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x)\right) \mid x\in X\right\} \quad (1)$$

where x is the discrete universe? The non-membership degree of any x is calculated by the subtraction $1-\mu_{\tilde{A}}(x)$.

The concept of a type-2 fuzzy set was introduced by Zadeh [2] as an extension of the concept of an ordinary fuzzy set. Such sets are fuzzy sets whose membership grades themselves are fuzzy. They are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. A type 2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type 2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as follows (Zadeh, 1975):

$$\tilde{\tilde{A}}=\left\{\left((x,u),\mu_{\tilde{\tilde{A}}}(x,u)\right) \mid \forall x\in X, \forall u\in XJ_x\subseteq[0,1], 0\le\mu_{\tilde{\tilde{A}}}(x,u)\le 1\right\}, \quad (2)$$

where J_x denotes an interval [0,1].

Intuitionistic fuzzy sets introduced by Atanassov [3] enable defining both the membership and non-membership degrees of an element in a fuzzy set. Their sum can be equal to or less than 1. The difference from 1, if any, is called hesitancy. Let U be a universe of discourse. An IFS $\tilde{I}$ is defined as follows:

Let X be a non-empty set. An intuitionistic fuzzy set I in X is given by:

$$I = \{(x, \mu_1(x), v_1(x)) \mid x \in X\} \quad (3)$$

where the function $\mu_I : X \to [0,1]$ and $\upsilon_I : X \to [0,1]$ defines the degree of membership and the degree of non-membership of element to the sets I, respectively, with the condition that.

$$0 \le \mu_1(x) + v_1(x) \le 1, for \forall x \in X \quad (4)$$

The degree of hesitancy is calculated as follows:

$$\pi_1(x) = 1 - \mu_1(x) - v_1(x) \quad (5)$$

Yager [4] first discussed fuzzy multi-sets, although he uses the term of fuzzy bag; an element of X may occur more than once with possibly the same or different membership values. Assume X is a set of elements. Then a fuzzy bag A drawn from X can be characterized by a function $Count.Mem_A$ such that

$$Count.Mem_A : X \to Q \quad (6)$$

where Q is the set of all crisp bags from the unit interval.

Smarandache [6] developed neutrosophic logic and neutrosophic sets (NSs) as an extension of intuitionistic fuzzy sets. The neutrosophic set is defined as the set where each element of the universe has a degree of truthiness, indeterminacy and falsity. The sum of these degrees can be at most equal to 3 since each of them can be independently at most equal to 1. Let E be a universe. A neutrosophic set $\tilde{\tilde{A}}$ in E is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A, and a falsity-membership function F_A.

$T_A(x), I_A(x)$ and $F_A(x)$ are real standart elements of [0,1]. A neutrosophic set $\tilde{\tilde{A}}$ can be given by Eq. (17):

$$\tilde{\tilde{A}} = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E, (T_A(x), I_A(x), F_A(x) \in]^-0, 1[^+ \quad (7)$$

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so that $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Nonstationary fuzzy sets are introduced by Garibaldi and Ozen [7]. Let A denote a fuzzy set of a universe of discourse X characterized by a membership function μ_A . Let T be a set of time points t_i (possibly infinite) and $f : T \to <$ denote the perturbation function. Associates with each element (t,x) of $T \times X$ a time specific variation of $\mu_A(x)$. The nonstationary fuzzy set $\dot{A}$ is denoted by

$$\dot{A} = \int_{t \in T} \int_{x \in E} \mu_{\dot{A}}(t, x) / x / t. \quad (8)$$

Hesitant fuzzy sets introduced by Torra [8] allow many potential degrees of membership of an element to be assigned to a set. These fuzzy sets force the membership degree of an element to be possible values between zero and one. A hesitant fuzzy set on X can be defined as in Eq. 9:

$$H = \{\langle X, h_H(x) \rangle \mid x \in X\} \quad (9)$$

where $h_H(x)$ is a set of hesitant fuzzy elements whose membership values are in [0,1].

Pythagorean fuzzy sets (PFSs) are an extension of intuitionistic fuzzy sets and it allows researchers to assign membership and non-membership degrees in a wider area. Atanassov's intuitionistic fuzzy sets of second type (IFS2) or Yager's Pythagorean fuzzy sets [9] are characterized by a membership degree and a non-membership degree satisfying that their squared sum is equal to or less than one, which is a generalization of intuitionistic fuzzy sets. This provides a larger area than IFS in order to assign membership and non-membership degrees [5]. Let ∪ be a universe of discourse. A PFS $\tilde{P}$ is an object having the form,

$$\tilde{P} = \{x, P(\mu_P(x), v_1(x)) \mid x \in X\} \quad (10)$$

where $\mu_P : X \to [0,1]$ is the membership degree and $v_P : X \to [0,1]$ is the non-membership degree. Then, Eq. (11) is valid:

$$(\mu_P(x))^2 + (v_1(x))^2 \le 1 \quad (11)$$

The degree of indeterminacy is defined as follows:

$$\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 - (v_P(x))^2} \quad (12)$$

IFS has been extended to picture fuzzy sets (PiFS). PiFS based approaches are more effective methods to meet different human views such as yes, abstain, no, and refusal. PiFS based models are successful in symbolizing uncertain information in different processes such as cluster analysis and pattern recognition. Cuong [10] introduced picture fuzzy sets (PiFS) which are direct extensions of intuitonistic fuzzy sets. A picture fuzzy set $\tilde{A}$ on the universe X is an object of the form.

$$\tilde{A} = \{\langle x; \mu_A(x), \eta_A(x), \vartheta_A(x), \rangle \mid x \in X\} \quad (13)$$

where $\mu_{\tilde{A}}(X) \in [0,1]$ is called the "degree of positive membership of $\tilde{A}$", $\eta_{\tilde{A}}(X) \in [0,1]$ is called the "degree of neutral membership of $\tilde{A}$" and $v_{\tilde{A}}(X) \in [0,1]$ is called the "degree of negative membership of $\tilde{A}$", and $\mu_{\tilde{A}}(x), \eta_{\tilde{A}}(x)$, and $v_{\tilde{A}}(x)$ satisfy the following condition: $0 \le \mu_{\tilde{A}}(x) + \eta_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1, \forall x \in X$. Then for $x \in X, \pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \eta_{\tilde{A}}(x) - v_{\tilde{A}}(x))$ could be called the degree of refusal membership of x in $\tilde{A}$. Thereafter, $\langle x; \mu_{\tilde{A}}(x), \eta_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x) \rangle$ will be given as $(x; \mu_{\tilde{A}}, \eta_{\tilde{A}}, \vartheta_{\tilde{A}})$.

Voting can be a good illustration of such a situation as the human voters may be divided into four groups of those who: vote for, hesitant, and vote against, refusal of the voting.

Q-Rung orthopair fuzzy sets (QROFSs) introduced by Yager [11] are represented with the degree of membership and non-membership. In q-ROFSs, the sum of the q^th power of the membership and non-membership degrees must be at most equal to one.

$$Q = \{(x, \mu_Q(x), v_Q(x)) \mid x \in X\} \quad (14)$$

where the function $\mu_Q : X \to [0,1]$ denotes the degree of membership and $\upsilon_Q : X \to [0,1]]$ denotes the degree of non-membership of the element $x \in X$ to the set Q, respectively, with the condition that $0 \le (\mu_Q(x))^q + (v_Q(x))^q \le 1, (q \ge 1)$ for every $x \in X$. The degree of indeterminacy is given as $\pi_P(x) = \sqrt[q]{1 - (\mu_Q(x))^q - (v_Q(x))^q}$.

When q = 3, Senapati and Yager [12] have called q-rung orthopair fuzzy sets as fermatean fuzzy sets (FFSs).

Let X be a universe of discourse. A Fermatean fuzzy sets F in X is an object having the form:

$$F = \{\langle x, \mu_F(x), v_F(x)\rangle \mid x \in X\} \quad (15)$$

where $\mu_F : X \to [0,1]$ and $\upsilon_F : X \to [0,1]$ which includes the circumstance

$$0 \le (\mu_F(x))^3 + (v_F(x))^3 \le 1 \quad (16)$$

for all $x \in X$. The numbers $\mu_F(x)$ and $v_F(x)$ indicate, respectively, the degree of membership and the degree of non-membership of the element x in the set F. For any FFS F and $x \in X$, the degree of hesitancy is calculated as follows:

$$\pi_F(X) = \sqrt[3]{1 - \mu_F(x)^3 - v_F(x)^3} \quad (17)$$

Spherical fuzzy sets (SFS) have been recently introduced by Gundogdu K, et al. [12]. These ets are based on the fact that the hesitancy of a decision maker can be assigned satisfying the condition that the squared sum of membership, non-membership and hesitancy degrees is at most equal to 1. In the following, definition of SFS is presented:

Single valued Spherical Fuzzy Sets (SFS) $\tilde{A}_S$ of the universe of discourse U is given by

$$\tilde{A}_S = \left\{\langle u, \left(\mu_{\tilde{A}_S}(u), v_{\tilde{A}_S}(u), \pi_{\tilde{A}_S}(u)\right) \mid u \in U\right\} \quad (18)$$

where

$$\mu_{\tilde{A}_S}(u) : U \to [0,1], v_{\tilde{A}_S}(u) : U \to [0,1], \pi_{\tilde{A}_S}(u) : U \to [0,1]$$

and

$$0 \le \mu^2_{\tilde{A}_S}(u) + v^2_{\tilde{A}_S}(u) + \pi^2_{\tilde{A}_S}(u) \le 1 \qquad \forall u \in U \quad (19)$$

For each u, the numbers $\mu_{\tilde{A}_S}(u), v_{\tilde{A}_S}(u)$ and $\pi_{\tilde{A}_S}(u)$ are the degree of membership, non-membership and hesitancy of to $\tilde{A}_S$, respectively.

Conclusion

A problem under imprecise and vague environment can be defined by fuzzy sets. More than 20 new extensions of fuzzy sets such as spherical fuzzy sets and circular intuitionistic fuzzy sets have been proposed in the literature for better expressing experts' thoughts by including additional parameters in membership and non-membership functions. It is observed that these parameters have been successfully used in modeling human thoughts. In this paper, it is observed that fuzzy sets and intelligence are widely used in today's technologies. In the future, it is thought that with the new extensions, use of fuzzy sets will increase with the integration of intelligence techniques.

References

1. Zadeh LA (1965) Fuzzy sets. Information Control 8(3): 338-353.
2. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning. Information sciences 8(3): 199-249.
3. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy sets and Systems 20(1): 87-96.
4. Yager RR (1986) On the theory of bags. International Journal of General System 13(1): 23-37.
5. Atanassov KT (1989) More on intuitionistic fuzzy sets. Fuzzy Sets and Systems 33(1): 37-45.
6. Smarandache F (1999) A unifying field in logics: Neutrosophic probability, set and logic. Rehoboth: American Research Press, Champaign, USA.
7. Garibaldi JM, Ozen T (2007) Uncertain fuzzy reasoning: A case study in modelling expert decision making. IEEE Transactions on Fuzzy Systems 15(1): 16-30.
8. Torra V (2010) Hesitant fuzzy sets. International Journal of Intelligent Systems 25(6): 529-539.
9. Yager RR (2013) Pythagorean fuzzy subsets. Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS). Edmonton, Canada.
10. Cuong BC (2014) Picture fuzzy sets. Journal of Computer Science and Cybernetics 30(4): 409-420.
11. Yager RR (2016) Generalized orthopair fuzzy sets. IEEE Trans. Fuzzy Syst 25(5): 1222-1230.
12. Senapati T, Yager TT (2019) Fermatean fuzzy sets. J Amb Intell Hum Comput 11: 663-674.
13. Gündoğdu K, Kahraman FC (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. Journal of Intelligent and Fuzzy Systems 36(1): 337-352.
14. Atanassov KT (2020) Circular intuitionistic fuzzy sets. Journal of Ambient Intelligence and Smart Environments 39(5): 5981-5986.