

Modified Holt's Linear Trend Method Based on Particle Swarm Optimization

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Abstract

Exponential smoothing methods have been commonly used for time series forecasting. Holt's linear trend exponential smoothing is a well-known exponential smoothing method and it can give successful forecasting results for time series which have trend component. In this study, a new modified Holt method is introduced. In modified Holt method, update formulas have second order lagged terms apart from classical Holt method. Moreover, initial values for trend and level and smoothing parameters are estimated by using particle swarm optimization. Strong and weak sides of the modified Holt method are investigated by using real-world data sets and simulated data sets.

Keywords: Forecasting; Exponential smoothing; Holt's linear trend method; Particle swarm optimization

Introduction

Time series analysis and forecasting are very important for many scientific disciplines. In the literature, various forecasting methods were proposed. Exponential smoothing methods constitute a class of classical forecasting methods. According to component of time series, it is possible to use different exponential smoothing methods. First studies on exponential smoothing are Brown et al. [1-5] made a classification on exponential smoothing methods. Box et al. [6-9] studies tried to find equivalence among exponential smoothing methods and Autoregressive Integrated Moving Average (ARIMA) models. Hyndman et al. [10] obtain confidence intervals for forecasts of exponential smoothing methods. Yapar G et al. [11] studies proposed modified exponential smoothing methods by using time variant smoothing parameters. In this study, a modified Holt's linear trend method is proposed. The proposed method uses second order update formulas apart from classical Holt's linear trend model. In addition to modified update formulas, the proposed method us particle swarm optimization to estimate smoothing parameters and initial values of trend and level. In the second section Holt's linear trend method is summarized. The brief information about Particle Swarm Optimization (PSO) is given in third section. In the fourth section the proposed method is introduced. The applications for real and simulated data sets are given in fifth section.

Holt's Linear Trend Method

Holt [3] modified simple exponential smoothing for trend component. This method is called Holt's linear trend method. In this method, level and trend of time series are estimated sample by sample in update formulas. Holt's linear trend method uses following formulas:

$$\text{Forecast Equation: } \hat{Z}_{n+1} = \hat{L}_n + \hat{b}_n \quad (1)$$

Level Equation:

$$\hat{L}_n = \lambda_1 \hat{Z}_n + (1 - \lambda_1) \hat{L}_{n-1} \quad (2)$$

Trend Equation:

$$\hat{b}_n = \lambda_2 (\hat{L}_n - \hat{L}_{n-1}) + (1 - \lambda_2) \hat{b}_{n-1} \quad (3)$$

Right sides of (2) and (3) contain first order lagged terms. For calculating forecasts by using (1), initial values of level and trend is needed. Initial values can be obtained by estimating following regression equation.

$$Z_t = \beta_0 + \beta_1 t + \varepsilon_t \quad (4)$$

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Least square estimations of β_0 and β_1 can be used as initial values of level and trend.

$$\hat{L}_0 = \hat{\beta}_0 \text{ and } \hat{b}_0 = \hat{\beta}_1 \quad (5)$$

Indeed, Holt's linear trend model is a kind of modification of (4) model. In (4) model level is fixed and trend is changing sample by sample using $\beta_1 t$. In (4) model, trend value is increased as $\hat{\beta}_1 t$ amount. In Holt's linear trend method, trend and level is changing sample by sample with equation (2) and (3). It is proved that forecasts of Holt's linear trend method are equal forecasts of second order differenced and second order autoregressive model (ARIMA (0,2,2)).

Particle Swarm Optimization

Estimating statistical model parameters is very important task. Statistical methods required strong optimization methods for maximizing likelihood or minimizing sum of error squares. For linear models, there is no need iterative or strong optimization methods because the estimators can be obtained by solving linear equation systems. Derivate based iterative algorithms are used for non-linear statistical models. These algorithms trap local minima for many of statistical model applications. For some statistical models, derivate cannot be calculated at some points. Artificial intelligence optimization techniques do not need derivate of objective function and it can be remedy for nonlinear statistical model estimation. PSO is a stochastic and swarm optimization technique. PSO algorithm do not need derivate of objective function and it can avoid local optimum trap. PSO is firstly proposed by Kennedy and Eberhart (1995). The algorithm of modified PSO is given below:

Algorithm 1.

Step 1. Generate the initial positions and velocities from uniform distribution.

$$P_i^{(0)} = \{p_{i,1}^{(0)}, p_{i,2}^{(0)}, \dots, p_{i,d}^{(0)}\}, i = 1, 2, \dots, pn \quad (6)$$

$$V_i^{(0)} = \{v_{i,1}^{(0)}, v_{i,2}^{(0)}, \dots, v_{i,d}^{(0)}\}, i = 1, 2, \dots, pn \quad (7)$$

$p_i^{(0)}$ and $v_i^{(0)}$ present positions and velocities of i th particle, respectively.

Step 2. According to initial positions of each particle, fitness functions are calculated.

Step 3. Determine P_{best} and g_{best}

$$P_{best}^{(t)} = \{p_{i,1}^{(t)}, p_{i,2}^{(t)}, \dots, p_{i,d}^{(t)}\}, i = 1, 2, \dots, pn \quad (8)$$

$$g_{best}^{(t)} = \{g_1^{(t)}, g_2^{(t)}, \dots, g_d^{(t)}\} \quad (9)$$

Step 4. Calculate new velocities and positions:

$$c_1^{(t)} = (c_{1f} - c_{1i}) \frac{t}{maxt} + c_{1i} \quad (10)$$

$$c_2^{(t)} = (c_{2f} - c_{2i}) \frac{t}{maxt} + c_{2i} \quad (11)$$

$$w_1^{(t)} = (w_2 - w_1) \frac{maxt-t}{maxt} + w_1 \quad (12)$$

$$v_{i,j}^{(t+1)} = w_1^{(t)} \times v_{i,j}^{(t)} + c_1^{(t)} \times rand_1 \times (p_{i,j}^{(t)} - v_{i,j}^{(t)}) + c_2^{(t)} \times rand_2 \times (g_j^{(t)} - v_{i,j}^{(t)}) \quad (13)$$

$$p_{i,j}^{(t+1)} = p_{i,j}^{(t)} + v_{i,j}^{(t+1)} \quad (14)$$

Step 5. Calculate the fitness function values for each particle.

Step 6. Update P_{best} and g_{best} . If the fitness value of g_{best} is smaller than pre-determined error tolerance, the training is ended for the bootstrap time series otherwise go to step 3,4.

Modified Holt's Linear Trend Method

Determining initial values of trend and level in Holt's linear trend method is important problem and it effects forecasting accuracy. Estimating smoothing parameters is other important task in Holt's linear trend method. A bound of Holt's linear trend method is using first order update formulas. In this study, it is focused on these three issues. A modified Holt's linear trend method is proposed [12]. The proposed method has new update equations for trend and level. Update equations of modified Holt's linear trend method are given below:

$$\text{Forecast Equation: } \hat{Z}_{n+1} = \lambda_1 (\hat{L}_n + \hat{b}_n) + (1-\lambda_1) (\hat{L}_{n-1} + \hat{b}_{n-1}) \quad (15)$$

$$\text{Level Equation: } \hat{L}_n = \lambda_2 (\lambda_3 Z_n + (1-\lambda_3) \hat{Z}_n) + (1-\lambda_2) (\lambda_4 Z_{n-1} + (1-\lambda_4) \hat{Z}_{n-1}) \quad (16)$$

$$\text{Trend Equation: } \hat{b}_n = \lambda_5 (\lambda_6 (\hat{L}_n - \hat{L}_{n-1}) + (1-\lambda_6) \hat{b}_{n-1}) + (1-\lambda_5) (\lambda_7 (\hat{L}_{n-1} - \hat{L}_{n-2}) + (1-\lambda_7) \hat{b}_{n-2}) \quad (17)$$

In the proposed method, $\lambda_i; i=0,1,2,\dots,7$ smoothing parameters and is needed \hat{L}_0 and \hat{b}_0 initial values are estimated by using PSO method. Each particle has 9 positions and they are presented in Table 1.

Table 1: Positions of a particle in the proposed method.

P1	P2	P3	P4	P5	P6	P7	P8	P9
\hat{L}_0	\hat{b}_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7

Initial values for first two positions of a particle is generated from normal distribution. Firstly, (4) regression model is applied to time series and parameters are estimated in the model. After that initial positions of first two positions are generated from $N(\hat{\beta}_0, 0.001)$ and $N(\hat{\beta}_1, 0.001)$ distributions. The initial positions of last seven positions are generated from uniform distribution with (0,1) parameters [13].

Applications and Simulation Results

The performance of the proposed method is investigated by designing a simulation study. In simulation study, time series data sets are simulated by using following equations:

$$X_t = 10 + 2t + \varepsilon_t \quad (18)$$

$$X_t = 10 + 2t + 2t^2 + \varepsilon_t \quad (19)$$

(18) is a linear trend, (19) is a quadratic trend model. The proposed method's performance is compared with Holt's linear trend method. In the generating series, the level of error variance

is taken as 10, 100 and 1000. The length of test set is takes as 10, 20 and 30. For each situation 100 time series are simulated. After simulated series are analyzed by the proposed method and Holt’s linear trend method, minimum and mean statistics of RMSE values for test set are computed for each situation and listed in Table 2 & 3.

Table 2: Simulation results according to minimum of RMSE statistics.

Simulation Model	ntest	Method	Sigma		
			10	100	1000
Linear	10	Holt M	4,95	50	661
		Proposed	4,41	53	701
	20	Holt M	7,38	72,27	807
		Proposed	6,24	72,5	726
	30	Holt M	7,86	69	777
		Proposed	5,69	79	752
Quadratic	10	Holt M	17	106	664
		Proposed	12	98	700
	20	Holt M	27	132	779
		Proposed	14	130	770
	30	Holt M	24	111	749
		Proposed	17	96	1000

Table 3: Simulation results according to mean RMSE statistics.

Simulation Model	ntest	Method	Sigma		
			10	100	1000
Linear	10	Holt M	10,69	101,1	1052,3
		Proposed	11,08	102,2	1033,8
	20	Holt M	10,42	103,2	1153
		Proposed	11,19	106,5	1119
	30	Holt M	10,43	109	1312
		Proposed	11,13	114	1237
Quadratic	10	Holt M	161	237	1221
		Proposed	125	227	1114
	20	Holt M	167	229	1227
		Proposed	116	219	1224
	30	Holt M	183	230	1516
		Proposed	109	249	1605

According to Table 2, the proposed method produced smaller RMSE value when the sigma level is 10. Moreover, the proposed method outperforms when the data generating process is quadratic trend model. According to Table 3 and mean statistics, the proposed method outperforms when the data generating process is quadratic trend model and smaller test set length. Secondly, Istanbul stock exchange data sets were analyzed by the proposed method, Autoregressive Integrated Moving Average Model (ARIMA), Multilayer Perceptron Artificial Neural Network (MLP-ANN) and Holt method. Istanbul stock exchange data sets is consisting of two

time series which are daily observed in between 01.02.2009 and 29.05.2009 and between 01.04.2010 and 05.31.2010. The last 7 and 15 observations are used as test sets for both time series. The obtained results for test sets are given in Table 4-7. When the Table 4-7 are examined the proposed method is outperforms others for 2009 year. The proposed method is the second-best method for 2010 year and ntest=7. The proposed method is still the best method for 2010 year and ntest=15.

Table 4: The results of ntest=7 for Istanbul stock exchange data sets between 01.02.2009 and 29.05.2009.

Date	Test Data	ARIMA	MLP-ANN	Holt M	Proposed M
5.21.2009	34721	35140	35179	35046	34571
5.22.2009	35015	34721	34803	34826	35014
5.25.2009	35408	35015	35114	35061	34987
5.26.2009	34861	35408	35407	35437	35329
5.27.2009	35169	34861	34966	34995	35219
5.28.2009	35021	35169	35246	35217	35129
5.29.2009	35003	35021	35098	35111	35196
	RMSE	344,91	325,1	310,09	259,1

Table 5: The results of ntest=15 for Istanbul stock exchange data sets between 01.02.2009 and 29.05.2009.

Date	Test Data	ARIMA	MLP-ANN	Holt M.	Proposed M.
5.8.2009	32806	32843	33061	32991	33239
5.11.2009	32203	32806	33159	32874	32916
5.12.2009	33043	32203	32531	32332	32507
5.13.2009	32829	33043	33339	32998	32853
5.14.2009	33095	32829	33031	32895	32999
5.15.2009	33485	33095	33356	33114	33101
5.18.2009	33666	33485	33651	33483	33449
5.20.2009	35140	33666	33795	33687	33701
5.21.2009	34721	35140	34926	35001	34733
5.22.2009	35015	34721	34547	34801	34961
5.25.2009	35408	35015	34887	35032	35011
5.26.2009	34861	35408	35108	35405	35371
5.27.2009	35169	34861	34727	34974	35145
5.28.2009	35021	35169	35002	35189	35161
5.29.2009	35003	35021	34845	35087	35169
	RMSE	540,2107	525,7	514,7464	498,389

Table 6: The results of ntest=7 for Istanbul stock exchange data sets between 01.04.2010 and 05.31.2010.

Date	Test Data	ARIMA	MLP-ANN	Holt M	Proposed M
5.21.2010	54112	54450	54249	54979	53928
5.24.2010	54558	54112	54460	54321	54288
5.25.2010	52257	54558	54751	54533	54150
5.26.2010	54104	52257	53127	52765	52140
5.27.2010	54498	54104	54013	53846	53963
5.28.2010	55234	54498	54715	54385	54441
5.31.2010	54385	55234	55032	55080	55197
	RMSE	1221	1077	1159	1141

Table 7: The results of ntest=15 for Istanbul stock exchange data sets between 01.04.2010 and 05.31.2010.

Date	Test Data	ARIMA	MLP-ANN	Holt M.	Proposed M.
5.10.2010	56448	52687	52671	52808	53629
5.11.2010	56462	56448	56424	56446	55602
5.12.2010	57976	56462	56438	56526	56595
5.13.2010	57930	57976	57902	58014	57717
5.14.2010	55748	57930	57859	57995	58079
5.17.2010	56071	55748	55718	55852	56456
5.18.2010	56978	56071	56045	56131	56124
5.20.2010	54450	56978	56949	57027	56878
5.21.2010	54112	54450	54398	54560	55249
5.24.2010	54558	54112	54058	54184	54338
5.25.2010	52257	54558	54507	54615	54579
5.26.2010	54104	52257	52275	52363	52996
5.27.2010	54498	54104	54050	54136	53758
5.28.2010	55234	54498	54446	54555	54532
5.31.2010	54385	55234	55195	55286	55179
	RMSE	1612	1603	1595	1471

Conclusion

In this paper, a modified Holt's linear trend method is proposed. The proposed method can determine initial values for trend and level. The proposed method uses second order update formulas, so it has better forecasting performance than classical Holt method. According to simulation study and real-world time series applications, the proposed method has smaller RMSE value for test sets than other alternative methods. In the future studies, the confidence intervals of the proposed method will be proved by using bootstrap techniques. Moreover, the similarity between ARIMA models and the proposed method will be examined.

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