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# Modified Holt's Linear Trend Method Based on Particle Swarm Optimization

**Egrioglu E[1]* and Baş W[2]**

[1]Department of Management Science, UK

[2]Department of Statistics,Turkey

**Abstract**

Exponential smoothing methods have been commonly used for time series forecasting. Holt's linear trend exponential smoothing is a well-known exponential smoothing method and it can give successful forecasting results for time series which have trend component. In this study, a new modified Holt method is introduced. In modified Holt method, update formulas have second order lagged terms apart from classical Holt method. Moreover, initial values for trend and level and smoothing parameters are estimated by using particle swarm optimization. Strong and weak sides of the modified Holt method are investigated by using real-world data sets and simulated data sets.

**Keywords:** Forecasting; Exponential smoothing; Holt's linear trend method; Particle swarm optimization


***Corresponding author:** Egrioglu E, Department of Management Science, UK

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## Introduction

Time series analysis and forecasting are very important for many scientific disciplines. In the literature, various forecasting methods were proposed. Exponential smoothing methods constitute a class of classical forecasting methods. According to component of time series, it is possible to use different exponential smoothing methods. First studies on exponential smoothing are Brown et al. [1-5] made a classification on exponential smoothing methods. Box et al. [6-9] studies tried to find equivalence among exponential smoothing methods and Autoregressive Integrated Moving Average (ARIMA) models. Hyndman et al. [10] obtain confidence intervals for forecasts of exponential smoothing methods. Yapar G et al. [11] studies proposed modified exponential smoothing methods by using time variant smoothing parameters. In this study, a modified Holt's linear trend method is proposed. The proposed method uses second order update formulas apart from classical Holt's linear trend model. In addition to modified update formulas, the proposed method us particle swarm optimization to estimate smoothing parameters and initial values of trend and level. In the second section Holt's linear trend method is summarized. The brief information about Particle Swarm Optimization (PSO) is given in third section. In the fourth section the proposed method is introduced. The applications for real and simulated data sets are given in fifth section.

## Holt's Linear Trend Method

Holt [3] modified simple exponential smoothing for trend component. This method is called Holt's linear trend method. In this method, level and trend of time series are estimated sample by sample in update formulas. Holt's linear trend method uses following formulas:

Forecast Equation: $\hat{Z}_{n+1} = \hat{L}_n + \hat{b}_n$ (1)

Level Equation: $\hat{L}_n = \lambda_1 Z_n + (1-\lambda_1)\hat{Z}_n$ (2)

Trend Equation: $\hat{b}_n = \lambda_2(\hat{L}_n - \hat{L}_{n-1}) + (1-\lambda_2)\hat{b}_{n-1}$ (3)

Right sides of (2) and (3) contain first order lagged terms. For calculating forecasts by using (1), initial values of level and trend is needed. Initial values can be obtained by estimating following regression equation.

$$Z_t = \beta_0 + \beta_1 t + \varepsilon_t \quad (4)$$

Least square estimations of $\beta_0$ and $\beta_1$ can be used as initial values of level and trend.

$$\hat{L}_0 = \hat{\beta}_0 \text{ and } \hat{b}_0 = \hat{\beta}_1 \quad (5)$$

Indeed, Holt's linear trend model is a kind of modification of (4) model. In (4) model level is fixed and trend is changing sample by sample using $\beta_1 t$. In (4) model, trend value is increased as $\hat{\beta}_{1t}$ amount. In Holt's linear trend method, trend and level is changing sample by sample with equation (2) and (3). It is proved that forecasts of Holt's linear trend method are equal forecasts of second order differenced and second order autoregressive model (ARIMA (0,2,2)).

## Particle Swarm Optimization

Estimating statistical model parameters is very important task. Statistical methods required strong optimization methods for maximizing likelihood or minimizing sum of error squares. For linear models, there is no need iterative or strong optimization methods because the estimators can be obtained by solving linear equation systems. Derivate based iterative algorithms are used for non-linear statistical models. These algorithms trap local minima for many of statistical model applications [12,13]. For some statistical models, derivate cannot be calculated at some points. Artificial intelligence optimization techniques do not need derivate of objective function and it can be remedy for nonlinear statistical model estimation. PSO is a stochastic and swarm optimization technique. PSO algorithm do not need derivate of objective function and it can avoid local optimum trap. PSO is firstly proposed by Kennedy and Eberhart (1995). The algorithm of modified PSO is given below:

Algorithm 1.

Step 1. Generate the initial positions and velocities from uniform distribution

$$P_i^{(0)} = \left\{ p_{i,1}^{(0)}, p_{i,2}^{(0)}, \ldots, p_{i,d}^{(0)} \right\}, i = 1, 2, \ldots, pn \quad (6)$$

$$V_i^{(0)} = \left\{ v_{i,1}^{(0)}, v_{i,2}^{(0)}, \ldots, v_{i,d}^{(0)} \right\}, i = 1, 2, \ldots, pn \quad (7)$$

$P_i^{(0)}$ and $V_i^{(0)}$ present positions and velocities of i[th] particle, respectively.

Step 2. According to initial positions of each particle, fitness functions are calculated.

Step 3. Determine $Pbest$ and $gbest$

$$Pbest_i^{(t)} = \left\{ Pb_{i,1}^{(t)}, Pb_{i,2}^{(t)}, \ldots, Pb_{i,d}^{(t)} \right\}, i = 1, 2, \ldots, pn \quad (8)$$

$$gbest^{(t)} = \left\{ g_1^{(t)}, g_2^{(t)}, \ldots, g_d^{(t)} \right\} \quad (9)$$

Step 4. Calculate new velocities and positions:

$$c_1^{(t)} = (c_{1f} - c_{1i})\frac{t}{maxt} + c_{1i} \quad (10)$$

$$c_2^{(t)} = (c_{2f} - c_{2i})\frac{t}{maxt} + c_{2i} \quad (11)$$

$$w_1^{(t)} = (w_2 - w_1)\frac{maxt - t}{maxt} + w_1 \quad (12)$$

$$v_{i,j}^{(t+1)} = w_1^{(t)} \times v_{i,j}^{(t)} + c_1^{(t)} \times rand_1 \times \left( Pb_{i,j}^{(t)} - p_{i,j}^{(t)} \right) + c_2^{(t)} \times rand_2 \times \left( g_j^{(t)} - p_{i,j}^{(t)} \right) \quad (13)$$

$$p_{i,j}^{(t+1)} = p_{i,j}^{(t)} + v_{i,j}^{(t+1)} \quad (14)$$

Step 5. Calculate the fitness function values for each particle.

Step 6. Update $Pbest$ and $gbest$. If the fitness value of $gbest$ is smaller than pre-determined error tolerance, the training is ended for the bootstrap time series otherwise go to step 3,4.

## Modified Holt's Linear Trend Method

Determining initial values of trend and level in Holt's linear trend method is important problem and it effects forecasting accuracy. Estimating smoothing parameters is other important task in Holt's linear trend method. A bound of Holt's linear trend method is using first order update formulas. In this study, it is focused on these three issues. A modified Holt's linear trend method is proposed. The proposed method has new update equations for trend and level. Update equations of modified Holt's linear trend method are given below:

Forecast Equation: $\hat{Z}_{n+1} = \lambda_1 \left( \hat{L}_n + \hat{b}_n \right) + (1 - \lambda_1) \left( \hat{L}_{n-1} + \hat{b}_{n-1} \right)$ (15)

Level Equation: $\hat{L}_n = \lambda_2 (\lambda_3 Z_n + (1 - \lambda_3) \hat{Z}_n) + (1 - \lambda_2)(\lambda_4 Z_{n-1} + (1 - \lambda_4) \hat{Z}_{n-1})$ (16)

Trend Equation: $\hat{b}_n = \lambda_5 (\lambda_6 (\hat{L}_n - \hat{L}_{n-1}) + (1 - \lambda_6) \hat{b}_{n-1}) + (1 - \lambda_5)(\lambda_7 (\hat{L}_{n-1} - \hat{L}_{n-2}) + (1 - \lambda_7) \hat{b}_{n-2}$ (17)

In the proposed method, $\lambda_i$; $i$=01,2,..,7 smoothing parameters and is needed $\hat{L}_0$ and $\hat{b}_0$ initial values are estimated by using PSO method. Each particle has 9 positions and they are presented in Table 1.

**Table 1:** Positions of a particle in the proposed method.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 |
|---|---|---|---|---|---|---|---|---|
| $\hat{L}_0$ | $\hat{b}_0$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ | $\lambda_7$ |

Initial values for first two positions of a particle is generated from normal distribution. Firstly, (4) regression model is applied to time series and parameters are estimated in the model. After that initial positions of first two positions are generated from $N(\hat{\beta}_0, 0.01)$ and $N(\hat{\beta}_1, 0.01)$ distributions. The initial positions of last seven positions are generated from uniform distribution with (0,1) parameters.

## Applications and Simulation Results

The performance of the proposed method is investigated by designing a simulation study. In simulation study, time series data sets are simulated by using following equations:

$$X_t = 10 + 2t + \varepsilon_t \quad (18)$$

$$X_t = 10 + 2t + 2t^2 + \varepsilon_t \quad (19)$$

(18) is a linear trend, (19) is a quadratic trend model. The proposed method's performance is compared with Holt's linear trend method. In the generating series, the level of error variance is taken as 10, 100 and 1000. The length of test set is takes as 10, 20 and 30. For each situation 100 time series are simulated. After simulated series are analyzed by the proposed method and Holt's linear trend method, minimum and mean statistics of RMSE values for test set are computed for each situation and listed in Table 2 & 3.

According to Table 2, the proposed method produced smaller RMSE value when the sigma level is 10. Moreover, the proposed method outperforms when the data generating process is quadratic trend model. According to Table 3 and mean statistics, the proposed method outperforms when the data generating process is quadratic trend model and smaller test set length. Secondly, Istanbul stock exchange data sets were analyzed by the proposed method, Autoregressive Integrated Moving Average Model (ARIMA), Multilayer Perceptron Artificial Neural Network (MLP-ANN) and Holt method. Istanbul stock exchange data sets is consisting of two time series which are daily observed in between 01.02.2009 and 29.05.2009 and between 01.04.2010 and 05.31.2010. The last 7 and 15 observations are used as test sets for both time series. The obtained results for test sets are given in Table 4-7. When the Table 4-7 are examined the proposed method is outperforms others for 2009 year. The proposed method is the second-best method for 2010 year and ntest=7. The proposed method is still the best method for 2010 year and ntest=15.

**Table 2:** Simulation results according to minimum of RMSE statistics.

| | | | Sigma | | |
|---|---|---|---|---|---|
| Simulation Model | N test | Method | 10 | 100 | 1000 |
| Linear | 10 | Holt M | 4,95 | 50 | 661 |
| | | Proposed | 4,41 | 53 | 701 |
| | 20 | Holt M | 7,38 | 72,27 | 807 |
| | | Proposed | 6,24 | 72,5 | 726 |
| | 30 | Holt M | 7,86 | 69 | 777 |
| | | Proposed | 5,69 | 79 | 752 |
| Quadratic | 10 | Holt M | 17 | 106 | 664 |
| | | Proposed | 12 | 98 | 700 |
| | 20 | Holt M | 27 | 132 | 779 |
| | | Proposed | 14 | 130 | 770 |
| | 30 | Holt M | 24 | 111 | 749 |
| | | Proposed | 17 | 96 | 1000 |

**Table 3:** Simulation results according to mean RMSE statistics.

| | | | Sigma | | |
|---|---|---|---|---|---|
| Simulation Model | N test | Method | 10 | 100 | 1000 |
| Linear | 10 | Holt M | 10,69 | 101,1 | 1052,3 |
| | | Proposed | 11,08 | 102,2 | 1033,8 |
| | 20 | Holt M | 10,42 | 103,2 | 1153 |
| | | Proposed | 11,19 | 106,5 | 1119 |
| | 30 | Holt M | 10,43 | 109 | 1312 |
| | | Proposed | 11,13 | 114 | 1237 |
| Quadratic | 10 | Holt M | 161 | 237 | 1221 |
| | | Proposed | 125 | 227 | 1114 |
| | 20 | Holt M | 167 | 229 | 1227 |
| | | Proposed | 116 | 219 | 1224 |
| | 30 | Holt M | 183 | 230 | 1516 |
| | | Proposed | 109 | 249 | 1605 |

**Table 4:** The results of n test=7 for Istanbul stock exchange data sets between 01.02.2009 and 29.05.2009.

| Date | Test Data | ARIMA | MLP-ANN | Holt M | Proposed M |
|---|---|---|---|---|---|
| 5.21.2009 | 34721 | 35140 | 35179 | 35046 | 34571 |
| 5.22.2009 | 35015 | 34721 | 34803 | 34826 | 35014 |
| 5.25.2009 | 35408 | 35015 | 35114 | 35061 | 34987 |
| 5.26.2009 | 34861 | 35408 | 35407 | 35437 | 35329 |
| 5.27.2009 | 35169 | 34861 | 34966 | 34995 | 35219 |
| 5.28.2009 | 35021 | 35169 | 35246 | 35217 | 35129 |
| 5.29.2009 | 35003 | 35021 | 35098 | 35111 | 35196 |
| | RMSE | 344,91 | 325,1 | 310,09 | 259,1 |

**Table 5:** The results of n test=15 for Istanbul stock exchange data sets between 01.02.2009 and 29.05.2009.

| Date | Test Data | ARIMA | MLP-ANN | Holt M. | Proposed M. |
|---|---|---|---|---|---|
| 5.8.2009 | 32806 | 32843 | 33061 | 32991 | 33239 |
| 5.11.2009 | 32203 | 32806 | 33159 | 32874 | 32916 |
| 5.12.2009 | 33043 | 32203 | 32531 | 32332 | 32507 |
| 5.13.2009 | 32829 | 33043 | 33339 | 32998 | 32853 |
| 5.14.2009 | 33095 | 32829 | 33031 | 32895 | 32999 |
| 5.15.2009 | 33485 | 33095 | 33356 | 33114 | 33101 |
| 5.18.2009 | 33666 | 33485 | 33651 | 33483 | 33449 |
| 5.20.2009 | 35140 | 33666 | 33795 | 33687 | 33701 |
| 5.21.2009 | 34721 | 35140 | 34926 | 35001 | 34733 |
| 5.22.2009 | 35015 | 34721 | 34547 | 34801 | 34961 |
| 5.25.2009 | 35408 | 35015 | 34887 | 35032 | 35011 |
| 5.26.2009 | 34861 | 35408 | 35108 | 35405 | 35371 |
| 5.27.2009 | 35169 | 34861 | 34727 | 34974 | 35145 |
| 5.28.2009 | 35021 | 35169 | 35002 | 35189 | 35161 |
| 5.29.2009 | 35003 | 35021 | 34845 | 35087 | 35169 |
| | RMSE | 540,2107 | 525,7 | 514,7464 | 498,389 |

**Table 6:** The results of n test=7 for Istanbul stock exchange data sets between 01.04.2010 and 05.31.2010.

| Date | Test Data | ARIMA | MLP-ANN | Holt M | Proposed M |
|---|---|---|---|---|---|
| 5.21.2010 | 54112 | 54450 | 54249 | 54979 | 53928 |
| 5.24.2010 | 54558 | 54112 | 54460 | 54321 | 54288 |
| 5.25.2010 | 52257 | 54558 | 54751 | 54533 | 54150 |
| 5.26.2010 | 54104 | 52257 | 53127 | 52765 | 52140 |
| 5.27.2010 | 54498 | 54104 | 54013 | 53846 | 53963 |
| 5.28.2010 | 55234 | 54498 | 54715 | 54385 | 54441 |
| 5.31.2010 | 54385 | 55234 | 55032 | 55080 | 55197 |
| | RMSE | 1221 | 1077 | 1159 | 1141 |

**Table 7:** The results of n test=15 for Istanbul stock exchange data sets between 01.04.2010 and 05.31.2010.

| Date | Test Data | ARIMA | MLP-ANN | Holt M. | Proposed M. |
|---|---|---|---|---|---|
| 5.10.2010 | 56448 | 52687 | 52671 | 52808 | 53629 |
| 5.11.2010 | 56462 | 56448 | 56424 | 56446 | 55602 |
| 5.12.2010 | 57976 | 56462 | 56438 | 56526 | 56595 |
| 5.13.2010 | 57930 | 57976 | 57902 | 58014 | 57717 |
| 5.14.2010 | 55748 | 57930 | 57859 | 57995 | 58079 |
| 5.17.2010 | 56071 | 55748 | 55718 | 55852 | 56456 |
| 5.18.2010 | 56978 | 56071 | 56045 | 56131 | 56124 |
| 5.20.2010 | 54450 | 56978 | 56949 | 57027 | 56878 |
| 5.21.2010 | 54112 | 54450 | 54398 | 54560 | 55249 |
| 5.24.2010 | 54558 | 54112 | 54058 | 54184 | 54338 |
| 5.25.2010 | 52257 | 54558 | 54507 | 54615 | 54579 |
| 5.26.2010 | 54104 | 52257 | 52275 | 52363 | 52996 |
| 5.27.2010 | 54498 | 54104 | 54050 | 54136 | 53758 |
| 5.28.2010 | 55234 | 54498 | 54446 | 54555 | 54532 |
| 5.31.2010 | 54385 | 55234 | 55195 | 55286 | 55179 |
| | RMSE | 1612 | 1603 | 1595 | 1471 |

## Conclusion

In this paper, a modified Holt's linear trend method is proposed. The proposed method can determine initial values for trend and level. The proposed method uses second order update formulas, so it has better forecasting performance than classical Holt method. According to simulation study and real-world time series applications, the proposed method has smaller RMSE value for test sets than other alternative methods. In the future studies, the confidence intervals of the proposed method will be proved by using bootstrap techniques. Moreover, the similarity between ARIMA models and the proposed method will be examined.

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