

Algebraic Analysis of q-Rung Orthopair Fuzzy Subgroup

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Abstract

q-rung orthopair fuzzy environment is the modern tool for handling uncertainty in many decisions making problems. In this manuscript, we use the concept of q-rung orthopair fuzzy subgroup. (q-ROPFSG) as a generalization of the Pythagorean fuzzy subgroup. We investigate various properties of our suggested fuzzy subgroup. Further, we define the notion of q-rung orthopair fuzzy group, q-ROPFSG and establish its related properties of it. Some theorems related to q-ROPFSG are proven.

Keywords: Fuzzy subgroup; Intuitionistic fuzzy subgroup; Pythagorean fuzzy subgroup; q-Rung Orthopairian group; q-Rung Orthopairian subgroup

Introduction

To handle uncertainty in real-life problems Zadeh LA [1] proposed the concept of Fuzzy Sets (FSs). In 1971 expanding the notion of FSs, Rosenfeld A [2] defined fuzzy subgroup. In recent years many researchers studied various properties of fuzzy subgroups, t-norm fuzzy subgroups, fuzzy level subgroup, fuzzy subgroups and fuzzy homomorphism, anti-fuzzy subgroups, fuzzy normal subgroup, fuzzy coset and fuzzy quotient subgroup etc [3-14]. In 1986, Atanassov KT [15] introduced Intuitionistic Fuzzy Set (IFS). In 1996, Intuitionistic fuzzy subgroup was first studied by Biswas R [16]. Zhan J et al. [17] introduced intuitionistic fuzzy M-group. Furthermore, researchers developed intuitionistic fuzzy subgroup in many ways [18-20]. In 2013, Yager RR [21] invented Pythagorean Fuzzy Sets (PFSs). In 2018, Naz S et al. [22] proposed a novel approach to decision-making problem using Pythagorean fuzzy set. In 2019, Akram M et al. [23] applied complex Pythagorean fuzzy set in decision-making problems. Ejegwa PA [24] gave an application of Pythagorean fuzzy set-in career placements based on academic performance using max-min-max composition. Some results related to it were given by Peng X [25] and Yang Y [26]. First time, Bhunia S et al. [27] proposed "On the characterization of Pythagorean fuzzy subgroups".

In certain situations, the PFSs unable to handle the problems like if membership degree $\eta = 0.8$ and nonmembership degree $\theta = 0.75$ then $\eta^2 + \theta^2 = 1.2025 \geq 1$ it is clear that PFSs fail to define this type of sets but, $\mu^q + \nu^q \leq 1$ where $q > 2$. Most recently another amazing generalization of FSs is proposed by Yager RR [28], q-Rung Orthopair Fuzzy Sets (q-ROFSs) which models uncertain and incomplete information better than both IFSs and PFSs with high accuracy. q-ROFSs is more fruitful in many decision-making problems. This concept is perfectly designed to represent imprecision and ambiguity in mathematical way and to produce a formalized tool to handle fuzziness to real problems. Pinar A et al. [29] suggested a q-rung orthopair fuzzy multi-criteria group decision making method for supplier selection based on a novel distance. Peng X et al. [30] published Information measures for q-rung orthopair fuzzy sets.

The rest of paper is organized as follow; Section 2 consists of some basic definition. In section, 3 we discussed q-rung orthopair fuzzy group, q-ROFSG and proof related theorems. Conclusion is stated in section 4.

Preliminaries

In this section, we review basic definitions of fuzzy subgroup, Intuitionistic fuzzy set, Intuitionistic fuzzy subgroup, Pythagorean fuzzy set, Pythagorean fuzzy subgroup and q-Rung orthopair fuzzy set. We represent membership degree with η , θ represent the nonmembership degree, π represent the degree of hesitancy and Θ shows binary operation.

Definition

[31]. Let $\eta: T \rightarrow [0,1]$ be fuzzy set and let (T, Θ) be a group. Then η is said to be fuzzy subgroup of (T, Θ) if the following axioms hold:

- (i) $\eta(p \Theta r) \geq \eta(p) \wedge \eta(r) \forall p, r \in T$,
- (ii) $\eta(p^{-1}) \geq \eta(p) \forall p \in T$,
- (iii) $\eta(e) = 1$,
- (iv) $\eta(p \Theta r) = \eta(r \Theta p)$.

Definition

[15]. An IFS \hat{T} in \hat{X} is define as

$$I = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$$

Where $\eta_I(p): T \rightarrow [0,1]$ represents the degree of membership and $\theta_I(p): T \rightarrow [0,1]$ the degree of non-membership of the element $p \in T$ to the set I with the condition that $0 \leq \eta(p) + \theta(p) \leq 1$. The degree of indeterminacy can be stated as $\pi_I(x) = 1 - \eta(p) - \theta(p)$.

Definition

[16]. Let $I = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$ be IFS and let (T, Θ) be a group. Then I is said to be intuitionistic fuzzy subgroup (IFSG) of (T, Θ) if the following axioms hold:

- (i) $\eta(p \Theta r) \geq \eta(p) \wedge \eta(r)$ and $\theta(p \Theta r) \leq \theta(p) \vee \theta(r) \forall p, r \in T$,
- (ii) $\eta(p^{-1}) \geq \eta(p)$ and $\theta(p^{-1}) \leq \theta(p) \forall p \in T$.

Definition

[21]. A Pythagorean fuzzy set (PFS) A in universal set is defined as

$$A = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$$

where $\eta_A(p): T \rightarrow [0,1]$ the membership and $\theta_A(p): T \rightarrow [0,1]$ is non-membership with the condition $0 \leq \eta^2(p) + \theta^2(p) \leq 1$. The hesitation degree of PFSs A is indicated $\pi_A(p)$ with the situation $\pi_A^2(p) \pi_A^2(p) = 1 - \eta^2(p) - \theta^2(p) \forall p \in T$.

Definition

[27]. Let $A = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$ be PFS and let (T, Θ) be a group. Then A is said to be Pythagorean fuzzy subgroup (PFSG) of (T, Θ) if the following axioms hold:

- (i) $\eta^q(p \Theta r) \geq \eta^q(p) \wedge \eta^q(r)$ and $\theta^q(p \Theta r) \leq \theta^q(p) \vee \theta^q(r) \forall p, r \in T$,
- (ii) $\eta^q(p^{-1}) \geq \eta^q(p)$ and $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$.

Definition

[28]. A q-ROFS in universal set is defined as

$$B = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$$

where $\eta_B(p): T \rightarrow [0,1]$ the membership and $\theta_B(p): T \rightarrow [0,1]$ is non-membership with the condition $0 \leq \eta^q(p) + \theta^q(p) \leq 1, q \geq 2$. The hesitation degree of q-ROFS B is indicated $\pi_B(p)$ with the situation $\pi_B^q(p) = 1 - \eta^q(p) - \theta^q(p), \forall p \in T$.

Definition

[29,30]. Assume A and B be two q-ROFSs, then the following operations can be defined as

- (i) $A \subseteq B$ iff $\eta_A(p) \leq \eta_B(p)$ and $\theta_A(p) \geq \theta_B(p)$;
- (ii) $A = B$ iff $\eta_A(p) = \eta_B(p)$ and $\theta_A(p) = \theta_B(p)$;
- (iii) $A \cup B = \{ \langle p, \eta_A(p) \vee \eta_B(p), \theta_A(p) \wedge \theta_B(p) \rangle : p \in T \}$;
- (iv) $A \cap B = \{ \langle p, \eta_A(p) \wedge \eta_B(p), \theta_A(p) \vee \theta_B(p) \rangle : p \in T \}$;
- (v) $A^c = \{ \langle P, \theta_A(p), \eta_A(p) \rangle : p \in T \}$.

q-Rung Orthopair Fuzzy Subgroup

Definition

Let $B = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$ be q-ROFSs and let (T, Θ) be a group. Then is said to be q-Rung Orthopair fuzzy group (q-ROFG) of (T, Θ) if the following axioms hold:

- (i) $\eta^q(p \Theta r) \geq \eta^q(p) \wedge \eta^q(r)$ and $\theta^q(p \Theta r) \leq \theta^q(p) \vee \theta^q(r) \forall p, r \in T$,
- (ii) $\eta^q(p \Theta (r \Theta s)) = \eta^q((p \Theta r) \Theta s)$ and $\theta^q(p \Theta (r \Theta s)) = \theta^q((p \Theta r) \Theta s) \forall p, r, s \in T$,
- (iii) $\eta^q(p^{-1}) \geq \eta^q(p)$ and $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$.
- (iv) $\eta^q(e) \geq \eta^q(p)$ and $\theta^q(e) \leq \theta^q(p) \forall p \in T$,
- (v) $\eta^q(p \Theta r) = \eta^q(r \Theta p), \forall r, p \in T$.

$$\eta^q_H(p) = \{ \eta(p) \}^q \text{ and } \theta^q_H(p) = \{ \theta(p) \}^q, \forall p \in T.$$

Definition

Let $B = \{ \langle p, \eta(p), \theta(p) \rangle \mid p \in T \}$ be q-ROFSs and let (T, Θ) be a group. Then B is said to be q-Rung Orthopair fuzzy subgroup (q-ROFSG) of (T, Θ) if the following axioms hold:

- (i) $\eta^q(p \Theta r) \geq \eta^q(p) \wedge \eta^q(r)$ and $\theta^q(p \Theta r) \leq \theta^q(p) \vee \theta^q(r) \forall p, r \in T$,
 - (ii) $\eta^q(p^{-1}) \geq \eta^q(p)$ and $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$.
- $$\eta^q_H(p) = \{ \eta(p) \}^q \text{ and } \theta^q_H(p) = \{ \theta(p) \}^q, \forall p \in T.$$

Example: Let us define q-ROFSG $T: P^2 = 1$. Show that (T, Θ) is a group w.r.t multiplication.

Solution. If $p^2 = 1$ then, $p = \pm 1$. We have to show that $T = \{-1, 1\}$ is q-ROFSG.

Let $\eta(1) = 0.8, \eta(-1) = 0.9$ and $\theta(1) = 0.3, \theta(-1) = 0.3$.
 Here, let $q = 3$ $\eta^3(1, (-1)) = \eta^3(-1) = (0.9)^3 = 0.729$
 and $\theta^3(1, (-1)) = \theta^3(-1) = (0.3)^3 = 0.027$ Now
 $\eta^3(1) \wedge \eta^3(-1) = \min\{\eta^3(1), \eta^3(-1)\} = \min\{0.512, 0.729\} = 0.512$
 and $\theta^3(1) \vee \theta^3(-1) = \max\{\theta^3(1), \theta^3(-1)\} = \max\{0.027, 0.027\} = 0.027$. So
 $\eta^3(1 \ominus -1) \geq \eta^3(1) \wedge \eta^3(-1)$ and $\theta^3(1 \ominus -1) \leq \theta^3(1) \vee \theta^3(-1)$. Hence is a
 q-ROFSG of the group (T, Θ) .

Example: Let us define q-ROFSG $T : P^4 = 1$. Show that (T, Θ) is a group w.r.t multiplication.

Solution. If $p^4 = 1$ then, $p = \pm 1, \pm i$. We have to show that $T = \{1, -1, i, -i\}$ is q-ROFSG.

Let $\eta(1) = 0.6, \eta(-1) = 0.7, \eta(i) = 0.8, \eta(-i) = 0.5$ and $\theta(1) = 0.4, \theta(-1) = 0.6,$
 $\theta(i) = 0.4, \theta(-i) = 0.6$. Here, $\eta^3(1, (-i)) = \eta^3(-i) = (0.5)^3 = 0.125$
 and $\theta^3(1, (-i)) = \theta^3(-i) = (0.6)^3 = 0.216$ Now
 $\eta^3(1) \wedge \eta^3(-i) = \min\{\eta^3(1), \eta^3(-i)\} = \min\{0.216, 0.125\} = 0.125$
 $\theta^3(1) \vee \theta^3(-i) = \max\{\theta^3(1), \theta^3(-i)\} = \max\{0.064, 0.216\} = 0.216$.

So $\eta^3(1 \ominus -i) \geq \eta^3(1) \wedge \eta^3(-i)$ and $\theta^3(1 \ominus -i) \leq \theta^3(1) \vee \theta^3(-i)$. Hence is a q-ROFSG of the group (T, Θ) .

In the same manner it can be shown that $\eta^q(p \Theta r) \geq \eta^q(p) \wedge \eta^q(r)$ $\theta^q(p \Theta r) \leq \theta^q(p) \vee \theta^q(r) \forall p, r \in T$, and approach the estimate results.

Proposition: Let (T, Θ) be a q-ROFSG. Then it holds all the following axioms;

- (i) $\eta^q(e) \geq \eta^q(p)$ and $\theta^q(e) \leq \theta^q(p), \forall p \in T$,
- (ii) $\eta^q(p^{-1}) \geq \eta^q(p)$ and $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$.

Proof. Let $\psi(\eta, \theta)$ is a q-ROFSG of a group (T, Θ) , then $\eta^q(p \Theta r) \geq \eta^q(p) \wedge \eta^q(r)$, $\theta^q(p \Theta r) \leq \theta^q(p) \vee \theta^q(r)$ and $\eta^q(p^{-1}) \geq \eta^q(p)$, $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$. Now $\eta^q(e) = \eta^q(p \Theta p^{-1}) \geq \eta^q(p) \wedge \eta^q(p^{-1}) = \eta^q(p)$. Also, $\theta^q(e) = \theta^q(p \Theta p^{-1}) \leq \theta^q(p) \vee \theta^q(p^{-1}) = \theta^q(p) \forall p \in T$. Hence, condition (i) is proved. Now, we have $\eta^q(p^{-1}) \geq \eta^q(p)$ and $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$. Putting p^{-1} in place of p , we have $\eta^q((p^{-1})^{-1}) \geq \eta^q(p^{-1}) \forall p \in T$ implies that $\eta^q(p) \geq \eta^q(p^{-1}), \forall p \in T$. Again, means that, $\theta^q(p) \leq \theta^q(p^{-1}), \forall p \in T$. Combining all these results we get $\eta^q(p) \geq \eta^q(p^{-1}), \theta^q(p) \leq \theta^q(p^{-1}), \forall p \in T$. Hence condition (ii) of proposition 3.1 is proved.

Proposition: Let (T, Θ) be a q-ROFSG. Then it holds associative property;

$$\eta^q(p \Theta (r \Theta s)) = \eta^q((p \Theta r) \Theta s) \text{ and } \theta^q(p \Theta (r \Theta s)) = \theta^q((p \Theta r) \Theta s) \forall p, r, s \in T.$$

Proof. Let $\psi(\eta, \theta)$ is a q-ROFSG of a group (T, Θ) , then $\eta^q(p \Theta (r \Theta s)) = \eta^q(p) \wedge \eta^q(r \Theta s) \geq \eta^q(p) \wedge \eta^q(r) \wedge \eta^q(s) = \min\{\eta^q(p), \eta^q(r), \eta^q(s)\}$, and similarly $\eta^q((p \Theta r) \Theta s) = \eta^q(p \Theta r) \wedge \eta^q(s) \geq \eta^q(p) \wedge \eta^q(r) \wedge \eta^q(s) = \min\{\eta^q(p), \eta^q(r), \eta^q(s)\}$, hence it is clear that $\eta^q(p \Theta (r \Theta s)) = \eta^q((p \Theta r) \Theta s), \forall p, r, s \in T$. Now, $\theta^q(p \Theta (r \Theta s)) \leq \theta^q(p) \vee \theta^q(r \Theta s) \leq \theta^q(p) \vee \theta^q(r) \vee \theta^q(s) = \max\{\theta^q(p), \theta^q(r), \theta^q(s)\}$ and $\theta^q((p \Theta r) \Theta s) \leq \theta^q(p \Theta r) \vee \theta^q(s) \leq \theta^q(p) \vee \theta^q(r) \vee \theta^q(s) = \max\{\theta^q(p), \theta^q(r), \theta^q(s)\}$. Hence it is clear that $\theta^q(p \Theta (r \Theta s)) = \theta^q((p \Theta r) \Theta s) \forall p, r, s \in T$.

Theorem: The subset H of a group T is a q-ROF subgroup of T it and only if

- (1) $\eta^q(p \Theta r) \geq \eta^q(p) \wedge \eta^q(r)$ and $\theta^q(p \Theta r) \leq \theta^q(p) \vee \theta^q(r) \forall p, r \in T$,
- (2) $\eta^q(p^{-1}) \geq \eta^q(p)$ and $\theta^q(p^{-1}) \leq \theta^q(p) \forall p \in T$,
- (3) $\eta^q(e) \geq \eta^q(p)$ and $\theta^q(e) \leq \theta^q(p), \forall p \in T$.

Proof. Let H be a subgroup of T . Then by definition H satisfies conditions (1), (2) and (3). Conversely, assume that conditions (1), (2) and (3) hold in H . We only need to show that H has associative property. We assume that $p, r, s \in H$, implies $p, r, s \in T$. Since $H \subseteq T$. Now $\eta^q(p \Theta (r \Theta s)) = \eta^q(p \Theta (r \Theta s))$ $\theta^q(p \Theta (r \Theta s)) = \theta^q((p \Theta r) \Theta s)$ exist in T . By condition (1) we have $\eta^q(p \Theta (r \Theta s)) \geq \eta^q(p) \wedge \eta^q(r \Theta s)$ and $\theta^q(p \Theta (r \Theta s)) \leq \theta^q(p) \vee \theta^q(r \Theta s)$, and $\eta^q(p \Theta (r \Theta s)) \geq \eta^q(p) \wedge \eta^q(r \Theta s) \geq \eta^q(p) \wedge \eta^q(r) \wedge \eta^q(s) = \min\{\eta^q(p), \eta^q(r), \eta^q(s)\}$, and similarly $\eta^q((p \Theta r) \Theta s) \geq \eta^q(p \Theta r) \wedge \eta^q(s) \geq \eta^q(p) \wedge \eta^q(r) \wedge \eta^q(s) = \min\{\eta^q(p), \eta^q(r), \eta^q(s)\}$, hence it is clear that $\eta^q(p \Theta (r \Theta s)) = \eta^q((p \Theta r) \Theta s), \forall p, r, s \in T$. Now, $\theta^q(p \Theta (r \Theta s)) \leq \theta^q(p) \vee \theta^q(r \Theta s) \leq \theta^q(p) \vee \theta^q(r) \vee \theta^q(s) = \max\{\theta^q(p), \theta^q(r), \theta^q(s)\}$ and $\theta^q((p \Theta r) \Theta s) \leq \theta^q(p \Theta r) \vee \theta^q(s) \leq \theta^q(p) \vee \theta^q(r) \vee \theta^q(s) = \max\{\theta^q(p), \theta^q(r), \theta^q(s)\}$. Hence it is clear that $\theta^q(p \Theta (r \Theta s)) = \theta^q((p \Theta r) \Theta s) \forall p, r, s \in T$.

Conclusion

In this paper, we introduce and explore the algebraic structure of q-rung orthopair fuzzy subgroups (q-ROFSG) as a generalization of intuitionistic and Pythagorean fuzzy subgroups. We establish several key definitions and prove various theorems that characterize the behavior and properties of these subgroups within fuzzy group theory. The findings reinforce the theoretical framework of fuzzy algebraic structures and demonstrate the enhanced flexibility of q-rung orthopair fuzzy sets in managing high levels of uncertainty, especially in cases where traditional fuzzy models fail. By providing detailed examples, we illustrated the validity of our proposed definitions and the applicability of q-ROFSG in algebraic settings. Furthermore, we identified the necessary and sufficient conditions under which a q-rung orthopair fuzzy group can be considered a q-rung orthopair fuzzy subgroup, offering a comprehensive foundation for further developments in this field.

Future Work

The present study lays the foundation for the algebraic analysis of q-rung orthopair fuzzy subgroups, but several avenues remain open for future exploration. One promising direction is to extend the proposed framework to q-rung orthopair fuzzy normal subgroups and investigate their properties under different group structures. Additionally, exploring the homomorphisms between q-rung orthopair fuzzy groups could provide deeper insight into structural transformations and their implications. Another potential area is the application of q-ROFSG in various real-life decision-making problems, particularly in fields such as artificial intelligence, data classification and pattern recognition, where uncertainty and imprecision play a crucial role. Further generalizations such as q-rung orthopair fuzzy cosets, quotient groups and their lattice structures can also be studied. These extensions would not only enrich the theoretical development but also broaden the applicability of q-rung orthopair fuzzy models in complex systems.

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