

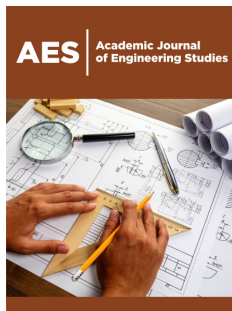
A Simplified Probability-Based likelihood Function for Bridge Condition Deterioration Model Updating

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Abstract

It is important to predict the future condition of bridges for bridge management. Statistical models of time-in-Condition (TC) can estimate the time that a bridge stays in a given condition, and then be used to predict the future condition of the bridge. To address the problem that a large number of bridge inspection data in China has not been fully recorded, a probability-based likelihood (PBL) function has been proposed in existing research to Bayesian update the TC models using incomplete inspection data. However, the existing PBL function contains multiple integrals, which is not easy to calculate and is not conducive to the engineering promotion of the method. In view of this situation, this paper proposes a simplified likelihood function by assuming that TCs follow independent normal distributions, which is an explicit formula that does not contain integral operations, and greatly reduces the calculation cost. Combined with Markov chain model, the formulas for the calculation of condition transition probabilities are derived based on current condition, time in current condition and service time in future. The accuracy of the proposed method is verified through numerical examples, and the effects of data amount, data completeness and data accuracy are explored.

Keywords: Bridge deterioration model; Condition rating; Bayesian updating; Time in condition; Inspection data

Introduction

Conditions of existing bridges gradually deteriorate over time, which may result in the safety of the bridges falling below the acceptable levels. For the prediction of long-term performance of a bridge, most BMS software today uses statistical models based on Condition Ratings (CR) of bridge components or of the entire bridge. The model to predict the future condition of bridges can be deterministic and generated by the method of regression [1] or artificial intelligence technology [2]. To consider the uncertainties in deterioration process, researchers used Markov chain to simulate the process that the CR of a bridge transfers due to deterioration [3]. Based on Markov chain model, inspection data were used to estimate the transition probability matrix, and then the distribution of the CR were calculated for future time [4,5]. The Markov chain model assumes that the transition probability does not change with time, and the future bridge condition depends only on the current bridge condition without considering the condition history, which would lead to inaccurate results.

Time-in-condition rating (TC) gives the length of time that a bridge stays in a specific condition. Deterioration models based on TC can capture the characteristics of bridge deterioration. Li & Jia [6] used probability density function directly as the likelihood function when estimating TC, which works only for cases where complete inspection records are available and the time interval between two consecutive inspections is short. To adapt to the current situation that the quality of existing bridge inspection data in China is poor, Zhang et al. [7] proposed a probability-based likelihood function for Bayesian updating of bridge condition deterioration model, which works for both cases of fully recorded inspection data

(referred to as “complete data”) and last inspection data only (referred to as “single data”). However, Zhang et al. [7] method involves complicated integral operations and is not convenient in engineering applications.

At present, the inspection data of many existing bridges in China are not well preserved, and it is not convenient to develop bridge condition deterioration model using existing methods. In view of this situation, this paper assumes the distribution of TC follow Normal distribution, and uses Bayesian updating method to estimate the parameters of TC distribution. A probability-based likelihood function is then proposed, which considers the randomness of condition changing time between two inspections, and has the characteristics of simple form and easy engineering application. Combined with the Markov chain model, the state transition probability expression based on the current condition, TC and subsequent service time is given. The accuracy of the method is verified through numerical examples, and the influence of bridge inspection data amount and data completeness on the accuracy of the model is discussed.

Condition Deterioration Model Based on Historical Inspection Data

Time-in-Condition (TC) model and Bayesian updating

A CR number is typically used to indicate the deterioration condition of a bridge, e.g., a number ranging from 1 to 5 is used for bridge CR in China [8], CR ranges from 0 to 9 in USA [9]. In this study, a CR of 1 represents the best condition (i.e., new bridges), and then the CR will gradually reduce over time. Statistical models of the TC assume that the length of time that a bridge stays in a specific condition is a random variable. Nasrollahi & Washer [10] indicated that Weibull distribution fits the data well, whose probability density function is:

$$f(x) = \frac{\alpha \cdot x^{\alpha-1}}{\theta^\alpha} \exp\left[-\left(\frac{x}{\theta}\right)^\alpha\right] \quad (1)$$

in which β is the shape parameter and θ is the scale parameter of the Weibull distribution.

To facilitate the engineering application, this paper assumes TC follow normal distribution, whose probability density function and cumulative distribution function are respectively:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad (2)$$

$$f(x) = \int_0^x \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx = \Phi\left(\frac{x-\mu}{\sigma}\right) \quad (3)$$

in which μ is the mean, σ is the standard deviation, and Φ is the cumulative function of normal distribution.

For an existing bridge, the historical bridge inspection data mainly includes the inspection time t_i of the i^{th} inspection of the bridge and the condition rating I_i evaluated based on the inspection results. Then the method of Bayesian updating is used to estimate the parameters of TC distribution. Let D represent the inspection data set, c represent the unknown parameter vector of

the model (here, the mean μ and the standard deviation σ of the distribution of TC), and $P(c)$ represent the prior distribution of the model parameter vector. According to Bayesian updating, the posterior distribution of the model parameter vector c is given by:

$$P(c|D) = \frac{P(c)P(D|c)}{P(D)} \quad (4)$$

in which $P(D)$ is the probability that the bridge condition data D would occur. $P(D|c)$ is the probability that the bridge condition data D would occur given the model parameter vector c , which is also known as the likelihood function. It is clear that for a given D , $P(D)$ is a constant, thus:

$$P(c|D) \propto P(c)P(D|c) \quad (5)$$

Since the prior distribution $P(c)$ is known, once the likelihood function $P(D|c)$ is obtained, the posterior distribution of the unknown parameter vector of the model $P(c|D)$ can be calculated.

Updating method of TC model

Since TC follows a normal distribution, with the mean and standard deviation being μ_j and σ_j for condition j , the time required for a bridge to enter condition i ($i > 1$), T_i , also follows a normal distribution, and its mean μ_{T_i} and standard deviation σ_{T_i} are:

$$\mu_{T_i} = \sum_{j=1}^{i-1} \mu_j, \quad \sigma_{T_i} = \sqrt{\sum_{j=1}^{i-1} \sigma_j^2} \quad (6)$$

Then the likelihood function of the bridge is the probability of the occurrence of current inspection result, i.e.,

$$P(D|c) = P(t \geq T_i \cap t < T_{i+1}) = \Phi\left(\frac{t - \mu_{T_{i+1}}}{\sigma_{T_{i+1}}}\right) - \Phi\left(\frac{t - \mu_{T_i}}{\sigma_{T_i}}\right) \quad (7)$$

For a bridge that has been inspected for many times, the inspection time for condition I is recorded as t_i . It may in the same condition for some of consecutive inspections; in which, only the first and the last of these inspections provide valuable information, and then only the first and the last inspection of the bridge at condition I will be used to calculate the likelihood function, recorded as $t_{I,\min}$ and $t_{I,\max}$, and other inspection data can be ignored. And the likelihood function of the bridge is given by:

$$P(D|c) = \Pi \left\{ \left[\Phi\left(\frac{t_{I,\max} - \mu_{I,i+1}}{\sigma_{I,i+1}}\right) - \Phi\left(\frac{t_{I,\max} - \mu_{I,i}}{\sigma_{I,i}}\right) \right] \left[\Phi\left(\frac{t_{I,\min} - \mu_{I,i+1}}{\sigma_{I,i+1}}\right) - \Phi\left(\frac{t_{I,\min} - \mu_{I,i}}{\sigma_{I,i}}\right) \right] \right\} \quad (8)$$

$$\Pi \left\{ \Phi\left(\frac{t_i - \mu_{I,i+1}}{\sigma_{I,i+1}}\right) - \Phi\left(\frac{t_i - \mu_{I,i}}{\sigma_{I,i}}\right) \right\}$$

in which the first $\Pi\{\}$ corresponds to the case of multiple inspection for one condition, the second $\Pi\{\}$ corresponds to the case of one inspection for one condition.

When one has data of multiple bridges, the overall likelihood function can be obtained by multiplying the likelihood functions of each bridge.

The Metropolis-Hasting Algorithm (MHA) is often applied to Markov chain Monte Carlo (MCMC) simulation and has recently been used in many engineering applications [11]. To get the posterior distribution of the model parameter vector c , MCMC simulation method with MHA is used to generate samples from $P(c)P(D|c)$. After generating sample distributions of $P(c)P(D|c)$, the mean values and confidence intervals of the parameters can be calculated.

Prediction of bridge condition deterioration based on TC model

Using the above Bayesian updating method, the posterior distribution parameters, μ_i and σ_i , of the TC_i of the bridge can be obtained. Knowing that TC actually conforms to the Weibull distribution, the parameters (scale factor θ_i and shape factor α_i) of the corresponding Weibull distribution are then calculated based on the mean and standard deviation,

$$\begin{aligned} \mu_i &= \theta_i \cdot \Gamma\left(1 + \frac{1}{\alpha_i}\right) \\ \sigma_i &= \theta_i \sqrt{\Gamma\left(1 + \frac{2}{\alpha_i}\right) - \Gamma^2\left(1 + \frac{1}{\alpha_i}\right)} \end{aligned} \tag{9}$$

in which Γ is the gamma function.

Then, the future degradation can be predicted based on the current condition of the bridge and the time it has been in that condition. For example, if the current CR of the bridge is I and it has been at that condition for a years, the probability of degrading to $CR(I+1)$ in the next year can be expressed as:

$$P_{I,I+1}(a,1) = \frac{\exp\left[-\left(\frac{a}{\theta_i}\right)^{\alpha_i}\right] - \exp\left[-\left(\frac{a+1}{\theta_i}\right)^{\alpha_i}\right]}{\exp\left[-\left(\frac{a}{\theta_i}\right)^{\alpha_i}\right]} = 1 - \exp\left[\left(\frac{a}{\theta_i}\right)^{\alpha_i} - \left(\frac{a+1}{\theta_i}\right)^{\alpha_i}\right] \tag{10}$$

It is a function of TC, a . As TC increases, the probability of deteriorating to the next state also increases.

Taking into account the change of state transition probability with TC, the probability of the bridge entering different degradation levels after several years can be further derived. Taking this bridge as an example, it has stayed in condition level I for a years. Then, after b years, the probability that the bridge is still in condition I is:

$$P_{I,I}(a,b) = \exp\left[\left(\frac{a}{\theta_i}\right)^{\alpha_i} - \left(\frac{a+b}{\theta_i}\right)^{\alpha_i}\right] \tag{11}$$

The probability of the bridge deteriorating to condition $(I+1)$ is considered in two cases,

① if condition $(I+1)$ is the worst condition, then:

$$P_{I,I+1}(a,b) = P(a < T_I \leq a+b < T_I + T_{I+1} | T_I > a) = \frac{\int_a^{a+b} \frac{\alpha_i x^{\alpha_i-1}}{\theta_i^{\alpha_i}} e^{-\left(\frac{x}{\theta_i}\right)^{\alpha_i}} e^{-\left(\frac{a+b-x}{\theta_{I+1}}\right)^{\alpha_{I+1}}} dx}{e^{-\left(\frac{a}{\theta_i}\right)^{\alpha_i}}} \tag{12}$$

② if condition $(I+1)$ is the worst condition, no deterioration will occur, that is, $T_{I+1} = \infty$, then the condition $a+b < T_I + T_{I+1}$ must be satisfied, and the probability is

$$P_{I,I+1}(a,b) = P(a < T_I \leq a+b | T_I > a) = \frac{e^{-\left(\frac{a}{\theta_i}\right)^{\alpha_i}} - e^{-\left(\frac{a+b}{\theta_i}\right)^{\alpha_i}}}{e^{-\left(\frac{a}{\theta_i}\right)^{\alpha_i}}} \tag{13}$$

The probability of the bridge deteriorating to condition $(I+2)$ is also divided into two cases:

① if condition $(I+2)$ is the worst condition, then:

$$\begin{aligned} P_{I,I+2}(a,b) &= P(a < T_I + T_{I+1} \leq a+b < T_I + T_{I+1} + T_{I+2} | T_I > a) \\ &= \frac{\int_a^{a+b} \frac{\alpha_i x^{\alpha_i-1}}{\theta_i^{\alpha_i}} e^{-\left(\frac{x}{\theta_i}\right)^{\alpha_i}} dx \int_0^{a+b-x} \frac{\alpha_{I+1} y^{\alpha_{I+1}-1}}{\theta_{I+1}^{\alpha_{I+1}}} e^{-\left(\frac{y}{\theta_{I+1}}\right)^{\alpha_{I+1}}} e^{-\left(\frac{a+b-x-y}{\theta_{I+2}}\right)^{\alpha_{I+2}}} dy}{e^{-\left(\frac{a}{\theta_i}\right)^{\alpha_i}}} \end{aligned} \tag{14}$$

② if condition $(I+2)$ is the worst condition, no deterioration will occur, that is, $T_{I+2} = \infty$, then the condition $a+b < T_I + T_{I+1} + T_{I+2}$ must be satisfied, and the probability is

$$P_{I,I+2}(a,b) = P(a < T_I + T_{I+1} \leq a+b | T_I > a) = \frac{\int_a^{a+b} \frac{\alpha_i x^{\alpha_i-1}}{\theta_i^{\alpha_i}} e^{-\left(\frac{x}{\theta_i}\right)^{\alpha_i}} \left(1 - e^{-\left(\frac{a+b-x}{\theta_{I+1}}\right)^{\alpha_{I+1}}}\right) dx}{e^{-\left(\frac{a}{\theta_i}\right)^{\alpha_i}}} \tag{15}$$

In China, when a bridge deteriorates to condition level 4, it is considered that some safety issues may arise and it needs to be repaired. Therefore, this study takes condition 4 as the worst level. Even if the current condition $I = 1$, $I+3$ is the worst condition, and the probability of deteriorating to this condition is

$$\begin{aligned} P_{I,I+3}(a,b) &= P(a < T_I + T_{I+1} + T_{I+2} \leq a+b | T_I > a) \\ &= \frac{\int_a^{a+b} \frac{\alpha_i x^{\alpha_i-1}}{\theta_i^{\alpha_i}} e^{-\left(\frac{x}{\theta_i}\right)^{\alpha_i}} dx \int_0^{a+b-x} \frac{\alpha_{I+1} y^{\alpha_{I+1}-1}}{\theta_{I+1}^{\alpha_{I+1}}} e^{-\left(\frac{y}{\theta_{I+1}}\right)^{\alpha_{I+1}}} e^{-\left(\frac{a+b-x-y}{\theta_{I+2}}\right)^{\alpha_{I+2}}} dy}{e^{-\left(\frac{a}{\theta_i}\right)^{\alpha_i}}} \end{aligned} \tag{16}$$

Illustration of the Method

To verify the accuracy of the proposed method, we first assume a known degradation model of the bridge condition, which provides the simulated inspection data for the proposed method, and also allows us to evaluate the accuracy of the method by comparing the “known model” with the analysis results [12].

Assume that for CR 1, 2 and 3, the distributions of TC follow Weibull distributions whose shape parameters are 2, 1.8 and 1.6 respectively, and the scale parameters are 20, 22 and 24 respectively. 50,000 bridge inspection data were then generated according to the Weibull distributions of CR 1, 2 and 3, and parts of the simulation results are shown in Figure 1.

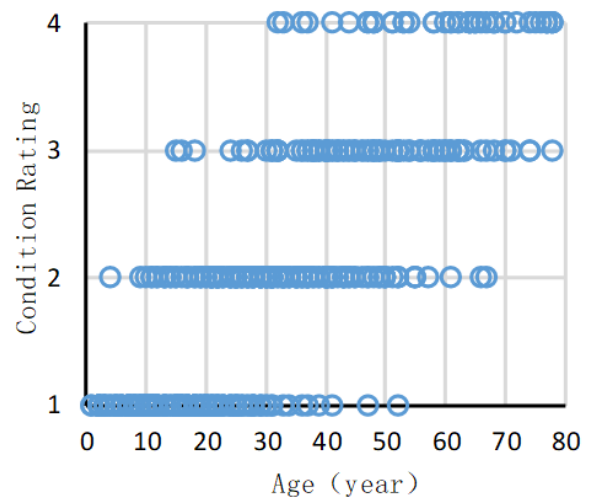


Figure 1: Simulated inspection results.

According to engineering experience, the TC of CR 1, 2, and 3 is generally between 5 and 30 years, and the variance is relatively large. Therefore, it is assumed that the means (μ_1 , μ_2 , μ_3) of the prior distribution of TC_1 , TC_2 , and TC_3 are uniformly distributed between 5 and 30, and the standard deviations (σ_1 , σ_2 , σ_3) are uniformly distributed between 5 and 15. The accepting samples are

obtained according to the method in Section 2.2, and some results are shown in Figure 2. Based on these samples, the mean values of the parameters ($\mu_1, \sigma_1, \mu_2, \sigma_2, \mu_3, \sigma_3$) are calculated, which

are: $\mu_1 = 17.68$ and $\sigma_1 = 9.94$; $\mu_2 = 19.72$ and $\sigma_2 = 10.47$; $\mu_3 = 20.74$ and $\sigma_3 = 11.17$.

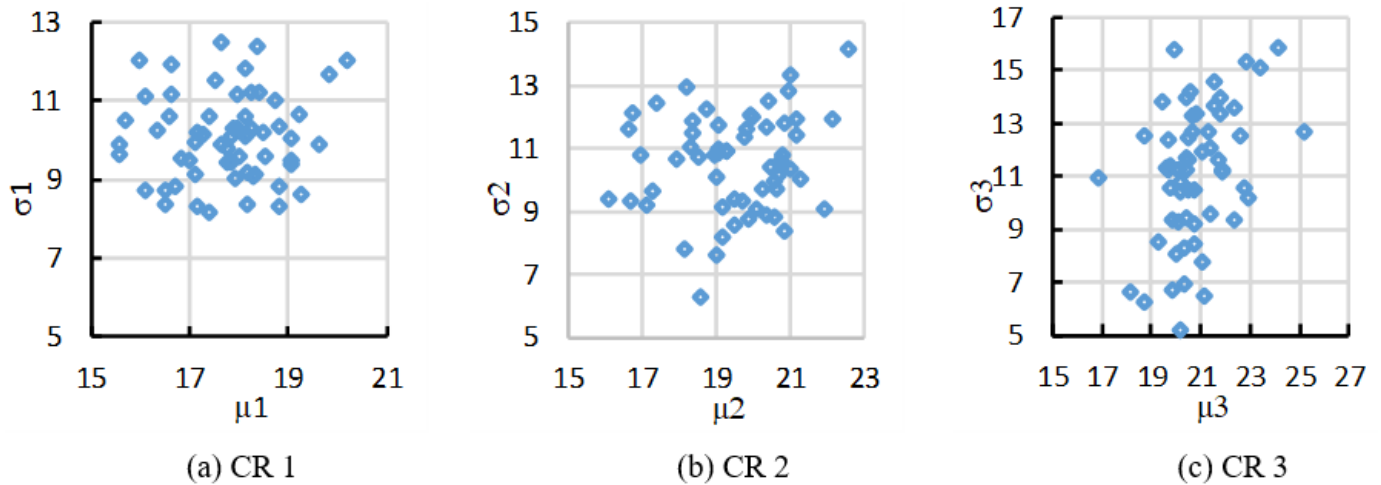


Figure 2: Some accepted samples of Bayesian updating for TC distributions of bridge.

Substituting the means and standard deviations of the three conditions into equation (11), the Weibull distribution parameters are obtained, which are $\theta_1 = 19.70$ and $\alpha_1 = 1.82$; $\theta_2 = 22.39$ and $\alpha_2 = 1.98$; $\theta_3 = 23.24$ and $\alpha_3 = 1.88$. And the probability distributions of TC_1, TC_2 and TC_3 are drawn and shown in Figure 3. The exact distributions are also plot in Figure 3. It can be seen that for the three conditions, the Bayesian updating results are all close to the exact results. In comparison, the update results of condition 1 are slightly better than those of condition 2, and the results of

condition 3 are the worst. This is because the bridge structure always gradually deteriorates from the better condition to the worse condition, and the data of the worse condition also contains the information about the better condition, so the estimation error of the better condition's TC will affect the estimation of the worse condition's TC. Therefore, the better the condition, the more information is available for it, and the more accurate the updating results are.

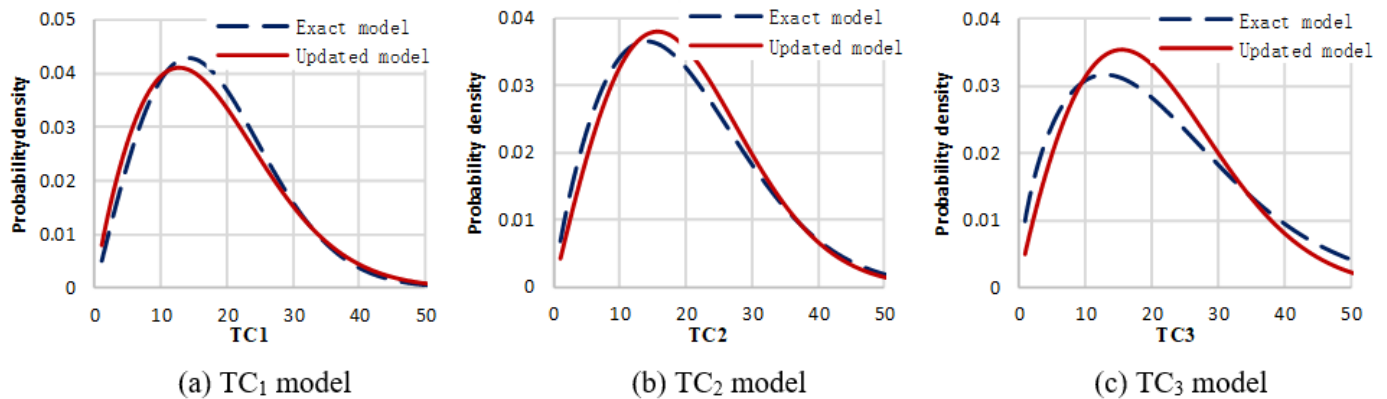


Figure 3: Comparison of probability distribution of TC between updated results and exact results.

With the updated TC models, the future deterioration of the bridge can be predicted. For example, the superstructures of bridges A and B are both in condition 1 for 0 years (Bridge A, new bridge) and 10 years (Bridge B) respectively, and the superstructures of bridges C and D are both in condition 2 for 0 years (Bridge C) and 10 years (Bridge D) respectively. As the service time increases, the probability distributions of the bridges in different conditions are calculated according to Equations. 11~16, and are shown in Figure 4. It is seen that, compared with the new bridge A, Bridge B is 10

years old. Although it is still in condition 1, its future deterioration will be significantly more serious than that of the new bridge A. According to the prediction results (Figure 4a & 4b), after 10 years, the probabilities of Bridge A being in conditions 1 to 3 are 0.206, 0.772, and 0.022, respectively; the probability of Bridge B remaining in condition 1 decreases to almost 0, the probability of deteriorating to condition 3 increases to 0.282, and it may deteriorate to condition 4 (the probability is 0.013).

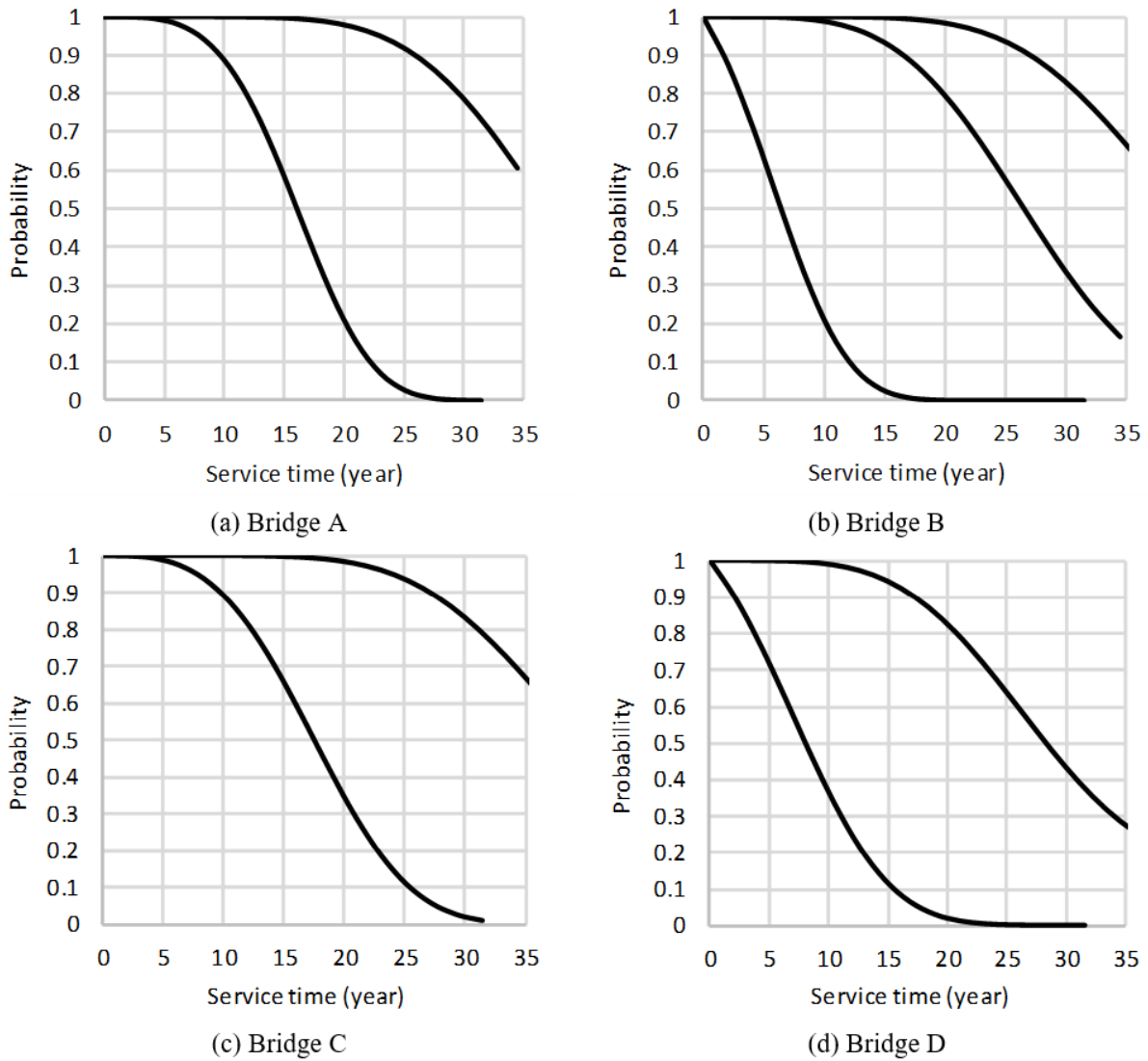


Figure 4: The probabilities of bridges in different conditions with the service time.

Since bridge C has just deteriorated to condition 2, according to the prediction results (Figure 4c & 4d), the probability that it will remain in this condition after 20 years is 0.334, the probability that it deteriorates to condition 4 is almost 0; while bridge D has deteriorated to condition 2 10 years ago, and the probability that it will still remain in this condition after 20 years is only 0.177, the probability of degenerating to conditions 3 and 4 increases to 0.637 and 0.186, respectively.

Discussion

In order to provide a reference for engineering applications, this section discusses the impact of data amount, data completeness, and data accuracy on the calculation accuracy of the proposed method.

Impact of data amount

The previous example used the inspection data of 500 bridges. Here, the number of bridges is increased to 1000 and reduced to 200 respectively. The TCs are estimated by the proposed method.

The updated probability distribution parameters of TCs are shown in Table 1. It can be seen that as the amount of inspection data increases, the updated TC models become more accurate.

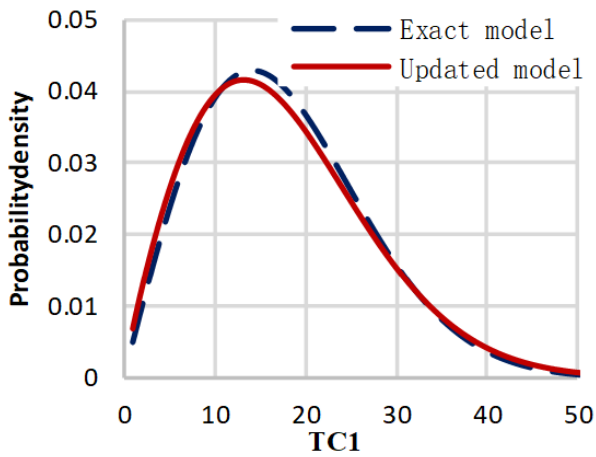
Table 1: Updated parameters using different amount of data.

Data amount	TC 1	TC 2	TC 3
200	$\alpha=1.80, \theta=19.6$	$\alpha=1.91, \theta=21.1$	$\alpha=2.25, \theta=22.6$
500	$\alpha=1.82, \theta=19.7$	$\alpha=1.98, \theta=22.4$	$\alpha=2.04, \theta=22.9$
1000	$\alpha=1.88, \theta=19.8$	$\alpha=1.96, \theta=22.3$	$\alpha=1.88, \theta=23.5$
Exact	$\alpha=2.0, \theta=20$	$\alpha=1.8, \theta=22$	$\alpha=1.6, \theta=24$

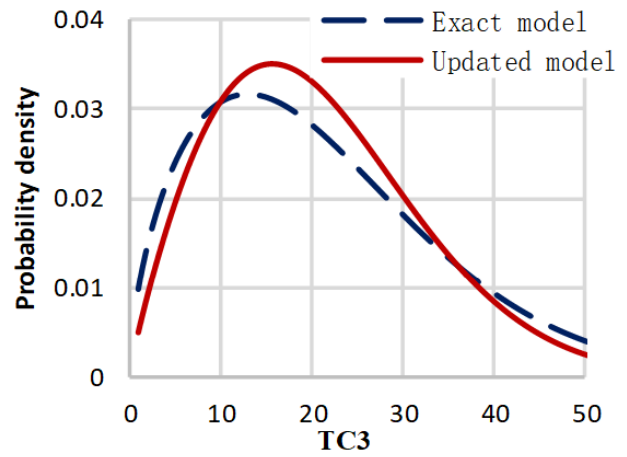
Figure 5 and Figure 6 respectively show the probability distribution of updated TC_1 and TC_3 model using the inspection data of 1000 and 200 bridges. It can be seen from Figure 5 that when the number of bridges increases from 500 to 1,000, the accuracy of TC_3 model is significantly improved, but the accuracy of TC_1 model is only slightly improved, which shows that when the bridge inspection data reaches a certain amount, the estimation error of

TC models comes from the replacement of Weibull distribution with the Normal distribution in the likelihood function. However, when the number of bridges is reduced from 500 to 200, the impact

of data amount is significant, as seen from Figure 6, the estimation accuracy of TC models is lower than that of the 500 and 1000 cases

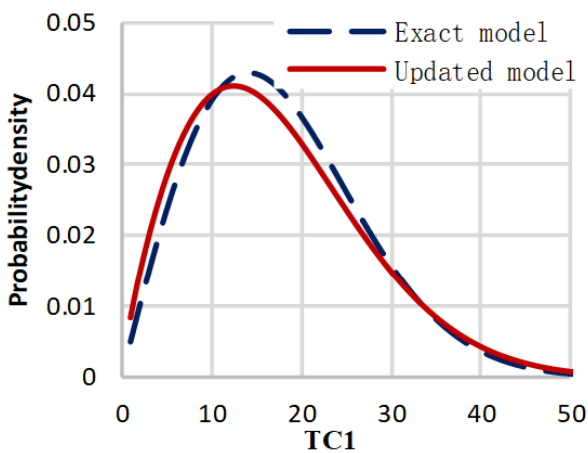


(a) TC model for condition 1

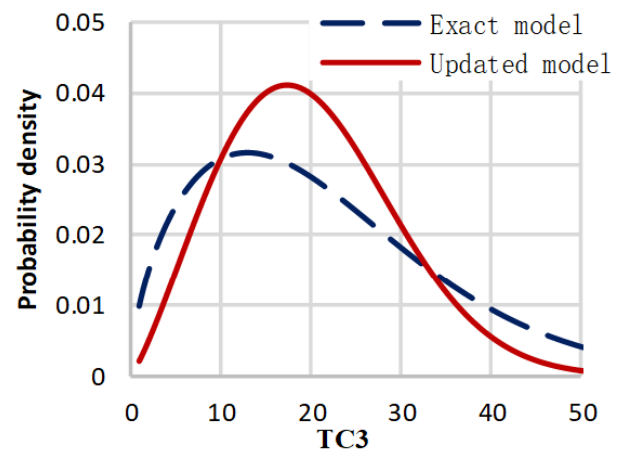


(b) TC model for condition 3

Figure 5: Updating results using inspection data of 1000 bridges.



(a) TC model for condition 1



(b) TC model for condition 3

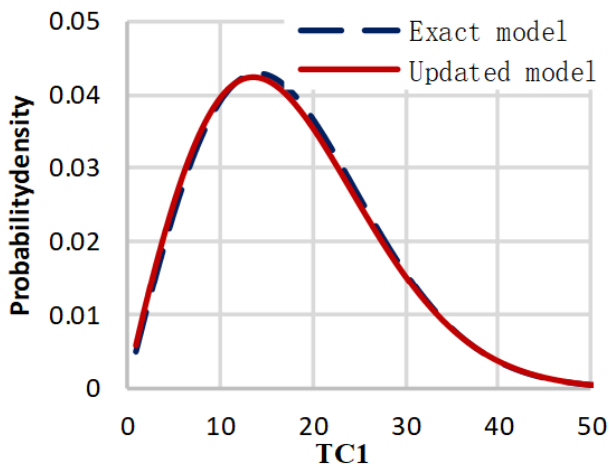
Figure 6: Updating results using inspection data of 200 bridges.

Impact of data completeness

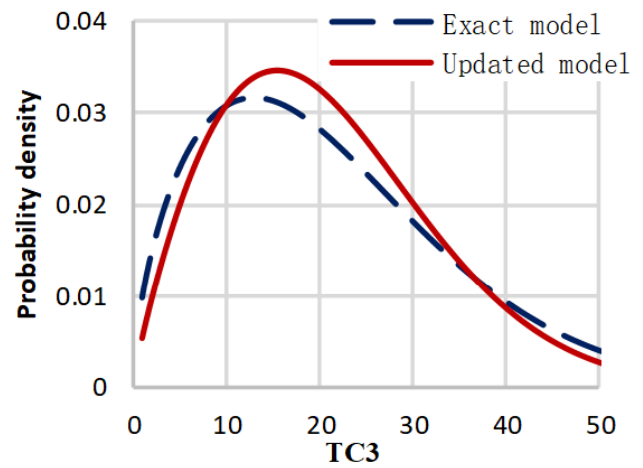
Data completeness indicates whether the bridge inspection history is complete and whether the data retention is complete. It is assumed that these 500 bridges are inspected once every two years and the historical records are kept complete. In this way, the data of each bridge includes the approximate time of entering and leaving all previous technical conditions in addition to the time of the last inspection and the condition level. Using the complete bridge inspection data, substituting them into the likelihood

function (Equation 8), through Bayesian updating, the probability distributions of TC_1 and TC_3 are obtained, as shown in Figure 7.

Compared with updated TC_1 and TC_3 in Figure 3, the results in Figure 7 are more accurate. This is because the complete data more accurately records the time of entering and leaving all technical conditions (from the beginning to the present), and the inspection record of each bridge contains multiple data for updating the time in condition.



(a) TC model for condition 1



(b) TC model for condition 3

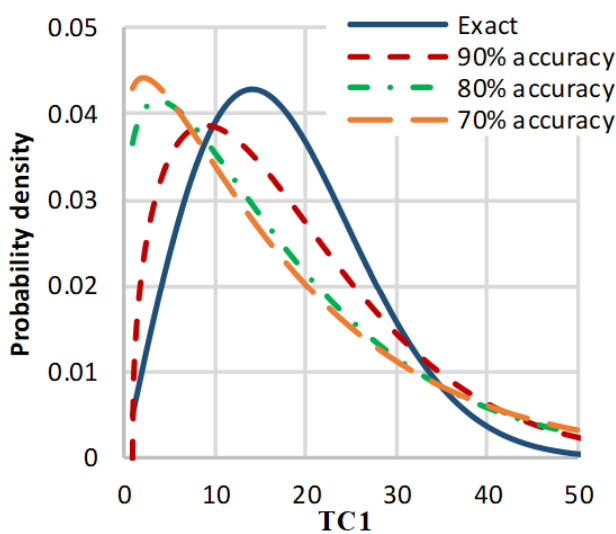
Figure 7: Updating results using complete inspection data.

Impact of data accuracy

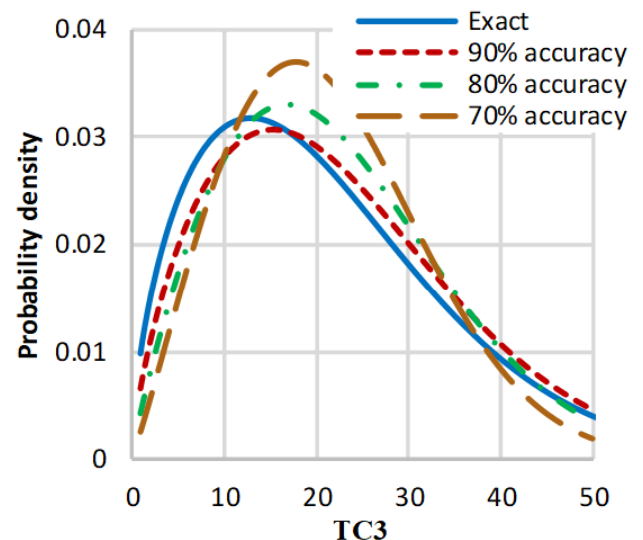
Due to the subjectivity of inspectors, the inspected CR may be higher or lower than the actual one, whose impact on updating accuracy is investigated herein. The single inspection data of 500 bridges are used, and 3 inspection accuracy rates are considered to reflect the subjectivity of inspectors, i.e., 90%, 80% and 70%. The accuracy rate of 90% assumes that 90% of the bridges are correctly rated, while 5% are rated one level higher and 5% are rated one level lower (for the case of the actual CR being 1, 95% are rated correctly, and 5% are rated as CR2). The impacts of inspection error on estimated TC_1 and TC_3 are shown in Figure 8, in which the results using accurate inspection data are also presented to make a comparison.

It can be seen from Figure 8 that as the inspection accuracy

rate decreases, the error associated with the updated TC models becomes more obvious. In particular, condition level 1 may be incorrectly assessed as condition level 2, resulting in the bridge inspection data underestimating the actual time in condition 1, which is a systematic bias in TC_1 estimate. When the inspection accuracy rate is 90%, the estimated TC_1 is about 3 years shorter than the actual; when the inspection accuracy is further reduced to 80% and 70%, the estimation error becomes more obvious. For condition 3, as the inspection accuracy rate decreases, the estimation error becomes larger too; however, when the inspection accuracy rate is above 80%, the updated results are still relatively accurate, and since there is no systematic deviation in bridge inspection data, the accuracy of the updating results can be further improved by increasing the amount of bridge inspection data.



(a) TC model for condition 1



(b) TC model for condition 3

Figure 8: Impact of inspection accuracy on updating accuracy.

Conclusion

A simplified probability-based likelihood function for Bayesian updating of time-in-condition of bridges is proposed in this paper. It is assumed that the distribution of TC follows a Normal distribution, and Bayesian updating based on Markov chain Monte Carlo is used to estimate the parameters of the distributions of TCs. Compared with existing methods, the main advantages of the proposed method are:

- (1) The method considers the discreteness of bridge inspection and the uncertainty of condition changing time between two inspections, so it can estimate the distribution of TCs accurately for both short and long inspection intervals.
- (2) The method can use both complete data and single data to estimate the distribution of TCs. Making full use of different kinds of data, the proposed method can get more accurate results.
- (3) The method is based on the assumption that TC follows Normal distribution. Although it brings certain error, the calculation is simple and easy for engineering application.

The accuracy of the method is affected by data amount, data completeness and data accuracy. To achieve a good accuracy, the number of bridges with single inspection data should be no less than 500, and the accuracy rate should be higher than 80%.

References

1. Hatami A and Morcouc G (2012) Deterioration models for life-cycle cost analysis of bridge decks in Nebraska. *Transportation Research Record* 2313(1): 3-11.
2. Ali G, Elsayegh A, Assaad R, Adaway EI (2020) Deck, superstructure, and substructure deterioration prediction for bridges using deep artificial neural networks. Paper presented at the Construction Research Congress (CRC) on Construction Research and Innovation to Transform Society, Arizona State Univ Del E Webb Sch Construct Tempe AZ, Arizona, USA.
3. Ranjith S, Setunge S, Venkatesan S, Samaratinga P (2009) Deterioration prediction of timber bridges using the Markov chain approach. 3rd Australian Small Bridges Conference. Small, Medium and Local Bridges, Sydney, Australia.
4. Liu H, Nehme J, Lu P (2023) An application of convolutional neural network for deterioration modeling of highway bridge components in the United States. *Structure and Infrastructure Engineering* 19(6): 731-744.
5. Miao PY, Yokota H (2022) Comparison of Markov chain and recurrent neural network in predicting bridge deterioration considering various factors. *Structure and Infrastructure Engineering* 20(2): 1-13.
6. Li M, Jia G (2020) Bayesian updating of bridge condition deterioration models using complete and incomplete inspection data. *Journal of Bridge Engineering* 25(3): 1-10.
7. Zhang Y, Quanwang L, Hao Z (2024) A probability-based likelihood function for bayesian updating of bridge condition deterioration model. *Journal of Bridge Engineering ASCE* 29(8): 04024054.
8. CJJ 99-2003 (2003) Technical Code of Maintenance for City Bridge. Ministry of Construction of the People's Republic of China, Beijing, China.
9. (1995) Recording and coding guide for the structure inventory and appraisal of the nation's bridges. Rep No. FHWA-PD-96-001. FHWA, Washington DC, USA.
10. Nasrollahi M, Washer G (2015) Estimating inspection intervals for bridges based on statistical analysis of national bridge inventory data. *Journal of Bridge Engineering* 20(9): 1-11.
11. Wellalage NW, Zhang T, Dwight R (2015) Calibrating Markov chain-based deterioration models for predicting future conditions of railway bridge elements. *Journal of Bridge Engineering* 20(2): 1-13.
12. Huang YH (2010) Artificial neural network model of bridge deterioration. *Journal of Performance of Constructed Facilities* 24(6): 597-602.