Stationary Poisson Process in Evaluation of HMA Specimens Fatigue Life

Andrzej Pozarycki*

Institute of Civil Engineering, Division of Road Engineering, Poznan University of Technology, Poland

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***Corresponding author:** Andrzej Pozarycki, Institute of Civil Engineering, Division of Road Engineering, Poznan University of Technology, Poznan, Poland

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Abstract

The fatigue life of hot mixed asphalt specimens was modelled in this paper as a complex modulus degradation process. In general, such a model consisting of random sum of random increments, provides the idea of using the stationary Poisson process. To modelling the constant-strain amplitude fatigue life test that is commonly known in pavement engineering practice as a four-point bending beam method, the cumulative jump model has been proposed. In order to estimate both the intensity and distribution parameters of Poisson process the random step curve of specimen complex modulus value degradation was de ned. It was shown that in case of developed experiment a relative error value for modelled fatigue life endurance is lower than 10% compared to the values resulting from the conventional fatigue life limit calculated as half of the initial value of complex modulus. Thus, the usefulness of the proposed method has been confirmed in the paper.

Keywords: Hot mixed asphalt; Beam specimen; Fatigue life; Cumulative jump; Stationary poisson process

Introduction

In the definition of limit strength conditions that include fatigue criteria of Hot Mixed Asphalt (HMA) specimens used for pavement layers of various types, one tries to control the crack development in the pavement structure. For ex- ample, the criterion of the classic study of Asphalt Institute (IA) indicates that if the level cracking in the wheel path pavement is ≥ 20% the limit of the conventional fatigue life is exceeded [1]. In order to describe fatigue phenomena, the techniques of crack propagation analysis and stiffness degradation are commonly used [2]. Regardless of the chosen method, however, the results of assessing fatigue phenomena of both HMA specimens [3] and other construction materials are of a very random character [4]. In general, to reduce the scatter of fatigue life, one can use a robust optimization model of structural fatigue life, which introduces the stochastic finite element method (SFEM) [5]. The formulation, as a multi-criteria optimization problem, in which both the mean structural fatigue life and the standard deviation of structural fatigue life are to be minimized, shows that the suggested method is feasible and applicable. There are also suggestions for multiple types of distributing the density of probability for the analysis of fatigue characteristics. In reference to fatigue life limit of asphalt concrete specimens, the proposed types are mainly log-normal [6], Weibull [7], or Beta-distribution. However, it is noteworthy that in the case of distribution of probability density, scattered results apply exclusively to empirical studies. At any rate, considering the stochastic analysis and the plastically induced fatigue damage provides more reliable results for fatigues life at the design state [8]. Nevertheless, it is a known fact that the basics of any description of phenomena of periodic nature (fatigue mechanisms) can be found in the theory of cracks development. In paper [4], the theory of stochastic processes is considered the most appropriate mathematical description of fatigue mechanisms. A fatigue crack growth is uncertain, either for the cracking rate or for the direction. A stochastic collocation method used for solving fatigue crack growth problems of mixed mode and with uncertain parameters was proposed in [9]. For dealing with fatigue life of materials, the performance has been evaluated on the basis of numerical examples, showing the usefulness for practical applications as well. In work [10], the authors described a Markov model that is utilized to predict the probability distribution of pavement crack depths with respect to the cumulative ESAL (Equivalent Single Axle Load) count. The model shows promise in terms of predicting the mean depth and the crack depth distributions. The use of the Markov model for a given service life was also introduced by [11]. A stochastic approach has been developed to estimate the required design thickness for flexible pavement using additional stochastic-based factors, which are mainly the initial and terminal transition probabilities. The formulation and analysis of response-degradation problems for randomly vibrating systems, coupled with stochastic dynamics, are presented in [12]. It is shown there that such an approach makes it possible to account for the effect of stiffness degradation on the response and simultaneously it yields the actual stress values for characterizing the accumulation of degradation.

The literature on the subject shows that the randomness in fatigue mechanisms of various materials can be interpreted and researched in many ways. Authors' own experience indicates that even during the experiments on fatigue mechanisms performed in laboratory conditions by following the standards EN 12697-24, "Bituminous mixtures test methods for hot mix asphalt Part 24: Resistance to fatigue", and with cyclic loads of constant amplitude, it is typical to observe a large scatter in the results of fatigue life [4]. In such cases, it is necessary to apply probabilistic methods [13]. However, in reference to the laboratory research on properties that determine the fatigue life of HMA specimens, methods based mainly on suggestions of different probability density functions (PDFs) are typically applied. Those distributions can, however, only be used for illustrating the scatter of results in empirical research. There are relatively few examples referring to this mechanism in terms of the theory of stochastic processes. Therefore, the main aim of this paper is a laboratory analysis of fatigue life of HMA in relation to probabilistic accumulation mechanisms of fatigue damage. The main body of the paper will be focused on the issues of estimating parameters for the cumulative model generated by a stationary Poisson process.

Methodology

The basis for the analysis is the laboratory test results concerning the determination of a conventional fatigue life of prismatic beams cut from the asphalt base layer. Using an analogy to the description of fatigue cracks propagation in materials [4], this paper adopts an assumption that a cumulative jump model in reference to fatigue test results includes the randomness of transition of a certain value from one state to the other. In reference to the fatigue life test results on HMA, the randomness of these changes is exposed through a probabilistic mechanism for changing the modulus value in i-th moment to the value in i + 1 moment, with the Poisson process intensity and through a probability density function of values of elementary drops of the complex modulus ($|E*|$).

Typical stiffness degradation in laboratory conditions

A typical example of degradation of the complex modulus values obtained from the fatigue life tests on asphalt concrete specimens in laboratory conditions has been presented in Figure 1a. The curves $|E*|(N)$ illustrate the process of degradation with the growing number of applied loads. The description concerns prismatic specimens cut from the asphalt base; 3-layer pavement of a test section described in paper [3]. Having in mind the conventional definition of the fatigue life criterion for HMA, the authors examined the specimens until the initial value of the complex modulus values (determined in 100th cycle) decreased by half. Figure 1 shows the results of the study of the conventional fatigue life limit of specimens submitted to symmetrical, harmonically varying loads of frequency amounting to 10 Hz and temperature of 15 °C. The research was performed in the conditions of indirectly controlled deformations amounting to $\epsilon=125 \cdot 10^{-6}$.

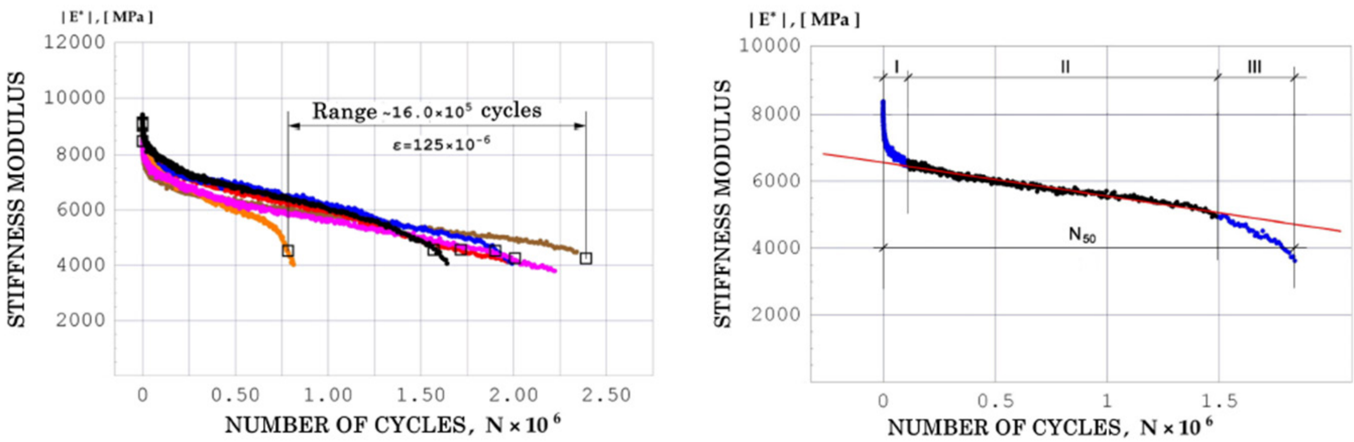

Figure 1: A graphic illustration of a typical degradation of complex modulus values in laboratory tests on specimens cut from the asphalt base, 3-layer pavement of a test section: a) results for 6 beam specimens submitted to cyclic loads causing permanent strain ϵ, b) typical degradation phases of complex modulus for a single specimen

The observed scatter of results, characteristic for HMA beam specimens submitted to fatigue life analysis, indicates a relatively high level of uncertainty of evaluation of conventional fatigue life endurance. A difference of $N_{max}-N_{min}$= 1.6 million cycles considerably limit the application of typically deterministic analysis of HMA fatigue life in this case.

Poisson cumulative jump process

The theory of stochastic processes [4] includes a group of mathematical methods that can also be applied in the description of fatigue mechanisms in hot mix asphalt. However, as of today this is not a standard solution. Having in mind that stochastic process (random process) is a function with random events assigned to its arguments, the authors show that this phenomenon can be constituted by the elements of fatigue degradation processes observed in the laboratory tests on specimens made of HMA.

The counting process considered in this paper is a stochastic process counting k events occurring before a certain moment. If the running time is assigned with a t variable (which assigns the initial moment with the value of 0), then, in general, the N (t) counting process is a function which:

A. Assumes the values from the set N0 = 0, 1, 2, ...

B. Is not decreasing

C. The N (t) − N (s) difference for the s < t is the number of events occurring within the time interval from the moment s to the moment t.

Such a N (t), t ≥ 0 counting process in which a) N (0) = 0, b) the number of events in two non-overlapping time intervals is independent, c) the number of events in a time interval of t length is given with Poisson distribution, is called a Poisson stochastic process with the intensity of λ > 0. For the range from 0 to t, this distribution is expressed with the relation (1).

$$N(t)\in N_0, N(0)=0 \ and \ P\{N(t)=k\}=\exp^{-\lambda t}\frac{(\lambda\cdot t)^k}{k!} \quad (1)$$

where: λ-the mean number of occurring events in a time unit (λ > 0), k = 0, 1, 2, ...

The process described with equation (1) is a special case of the inhomogeneous Poisson process in which the λ(t) intensity parameter is not time-dependent.

This means that its value is constant, and the process itself is called a homogeneous Poisson process.

HMA specimen complex modulus values degradation curve

While considering an empirical curve |E∗|(N) shown in Figure 2, it can be concluded that the process of "losing stiffness" by a beam specimen with the increasing number of loading cycles is discrete (stepwise). Using the condition given by inequality (2) one can indicate only filtered values, which in consequence refer to a random step curve shown in Figure 2b.

$$E_{i+1}-E_i<0 \quad (2)$$

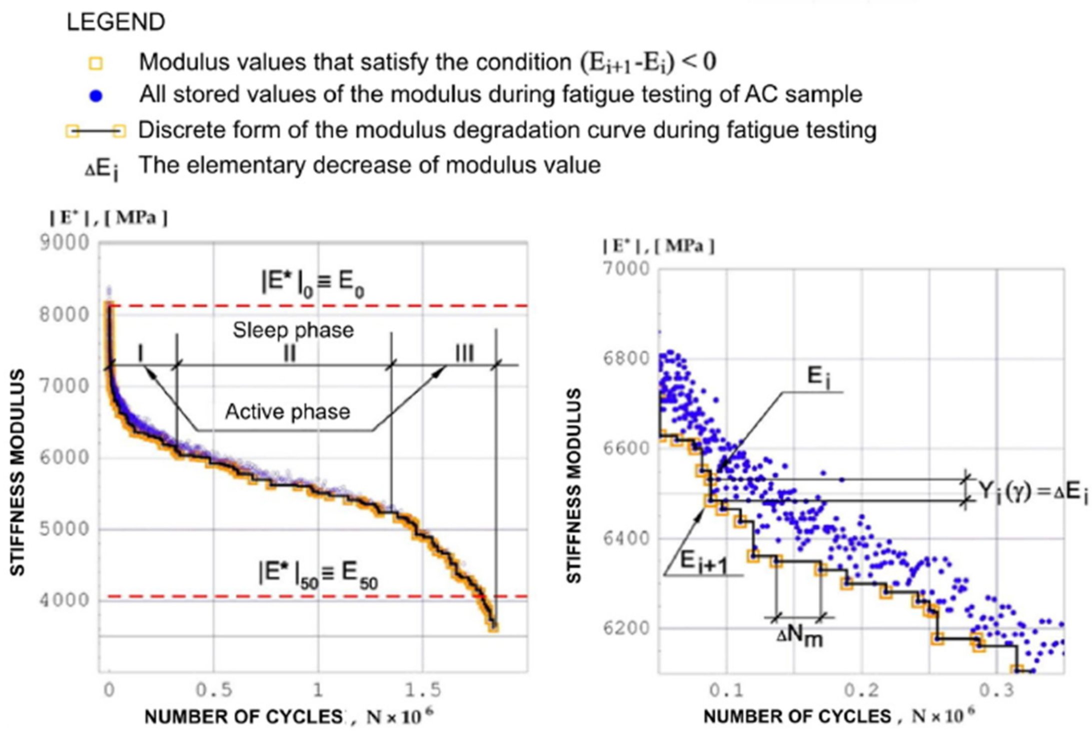

Figure 2: A graphic illustration of a step curve describing the discrete degradation form of the complex modulus values during the laboratory fatigue tests.

The paper assumes that a curve created in this manner can be treated as a discrete and random model of the fatigue degradation process of the HMA specimens.

A cumulative jump model generated by poisson process

Degradation of complex modulus values. Formulating the properties of the |E∗|(N) curve (or analogously |E∗|(t) in the time domain) in the form shown in Figure 2 allows to assume that the value of the complex modulus in fatigue tests decreases in a stepwise manner with the increasing number of loading cycles (a complex modulus submits to fatigue degradation). In any moment, this value is a random variable, and the distribution of density of probability (CDF cumulative distribution function) is expressed by the relation (3).

$$F_E(u;t) = P\{E(t) \le u\}, \quad E(t) = f(|E^*|(N)) \quad (3)$$

where:

u-the typical (conventional) critical value of the complex modulus.

The process of fatigue degradation of the values of complex modulus in time can be written as a random sum of random elements (4).

$$E(t,\gamma) = E_0 - E_1(t,\gamma) = E_0 - \sum_{i=1}^{N(t)} Y_i(\gamma),\ Y_i(\gamma) = \Delta E_i \quad (4)$$

where:

E0-the initial, deterministic value of the complex modulus determined based on the tests (here 4PBB method)

γ-elementary drops of the complex modulus values in the ti moments

Yi(γ)-random variables describing elementary drops of the complex modulus values in the ti moments

N (t) -the counting process describing the number of real, elementary value drops of the modulus in the time interval from 0 to t (natural numbers).

Assuming a priori that the random ΔEi variable is approximated with the exponential distribution, one can derive a formula for the function of density of probability for the complex modulus in any t moment fE1 (u; t) [3,4]. The final form of this relation is expressed by equation 5.

$$f_{E_1}(u;t) = e^{-\lambda.t-\alpha.u} \sum_{k=0}^{\infty} \frac{(\alpha\cdot\lambda\cdot t)^{k+1}}{(k+1)!} \frac{u^k}{k!} \quad for\, u > 0 \quad (5)$$

where:

α-parameter of random ΔEi variable distribution

u = E0/2.

The remaining symbols have been already explained earlier.

Fatigue life description. When considering time in which the value of the complex modulus of a specimen decreases by half in comparison to its initial value (it will reach the conventional critical ξ value), one needs to analyze the prob- ability de ned with formula (6).

$$P\{T > t\} = P\{E(t,\gamma) < \xi\} \quad (6)$$

where:

E(t, γ)-fatigue degradation process of the value of the modulus of the HMA specimen

ξ-the critical value of the fatigue life. This value is a fatigue life definition- based decision, and here it was assumed that the HMA specimen reaches the critical fatigue life limit in moment when its complex modulus decreased by half, that is ξ = E0/2

t-time after which the initial value decreases by half as a result of material fatigue.

Similarly to the case of complex modulus analysis, it is a prior assumed that the density of probability of elementary value drops of complex moduli can also be described by the exponential distribution. In consequence, this leads to a relation (7) expressing the form of a random variable describing the searched value of conventional time of fatigue fT (t).

$$f_T(t) = \lambda\cdot e^{-\lambda\cdot t-\alpha(E_0-\xi)} \sum_{k=0}^{\infty} \frac{(\lambda\cdot t)^k}{k!} \frac{\alpha^k (E_0-\xi)^k}{k!} \quad (7)$$

where:

E0- known initial value of the complex modulus of the HMA specimen.

All of the rest of symbols used in relation (7) have been successively explained before.

Research Experiment

The application of fatigue degradation description is presented in reference to the results of fatigue tests shown in Figure 1. The presentation of the detailed analysis has been restricted to the description of the properties of the curve shown in Figure 1b.

The intensity of Poisson process, λ parameter

First and foremost, one should note that the specific value jumps of the ΔEi complex modulus which form the shape of the step curve can be considered k events describing a certain random variable. A task formulated in this manner means that in reference to laboratory results of HMA fatigue tests, a random configuration of these events is the property of the process of a gradual value degradation of a complex modulus in the function of time (or alternatively, in the function of fatigue cycles number) (Figure 3).

Further deduction in this direction leads to the solution in which an approximation of the value of intensity parameter can be reduced to determining the number of events in the unit of time. By defining the moment of losing endurance on the basis of fatigue test results, it was concluded that the intensity of events can be expressed with relation (8).

$$\lambda = \frac{k}{\bar{t}} \qquad (8)$$

It is also noteworthy that, in the considered time interval, intensity is not a parameter with a constant value. Treating this value as constant is a simplification assumed for the reason of applying a cumulative model generated by a homogeneous Poisson process. In general, the intensity graph for specific phases, in confrontation with the intensity calculated according to formula (8) for the analyses sample of hot mix asphalt, is shown in Figure 4. It is a representation of a typical situation characteristic for the remaining specimens as well, whose fatigue test results can be seen in Figure 1.

In the following section of the paper, for calculating λ value the authors considered the whole lifetime interval of the hot mix asphalt sample, that is, from 0 moment to the conventional fatigue limit equal to $\bar{t}$.

Random ΔEi variable distribution, parameter α

In reference to the issues discussed here, in the cumulative jump model generated by a stationary Poisson process, assuming that the distribution of elementary ΔEi values is an exponential distribution, one needs to estimate its value. Unlike the method based on the mean squared criterion [4], the current paper presents a hypothesis that a certain approximation of the form of the probability density function of elementary changes (jumps) of modulus values (ΔEi, Figure 3), from the beginning of the test until the conventional t destruction, can be a distribution based on the values of those jumps resulting from a specific form of an empirical curve of fatigue degradation. Therefore, one should consider all value jumps for the ΔEi modulus occurring from the moment when the sample had properties of an initial complex modulus until the moment of conventional fatigue. Consequently, it is accepted that in a cumulative model generated by the Poisson process, the probability density function describing value jumps of a modulus within the $(0 \div \bar{t})$ time interval is close to the probability density function describing the elementary jumps of the complex modulus registered during the fatigue tests performed on the specimens made of hot mix asphalt. Detailed results of statistical calculations indicate that it is possible to take into account two theoretical functions of density of probability, which, in a satisfyingly accurate manner (in the mean of the χ2 test) characterize the distribution of those values. Those PDFs are given with exponential and gamma functions. The form of those functions for the population of specimens cut from the asphalt pavement base of the test section is presented in Figure 5.

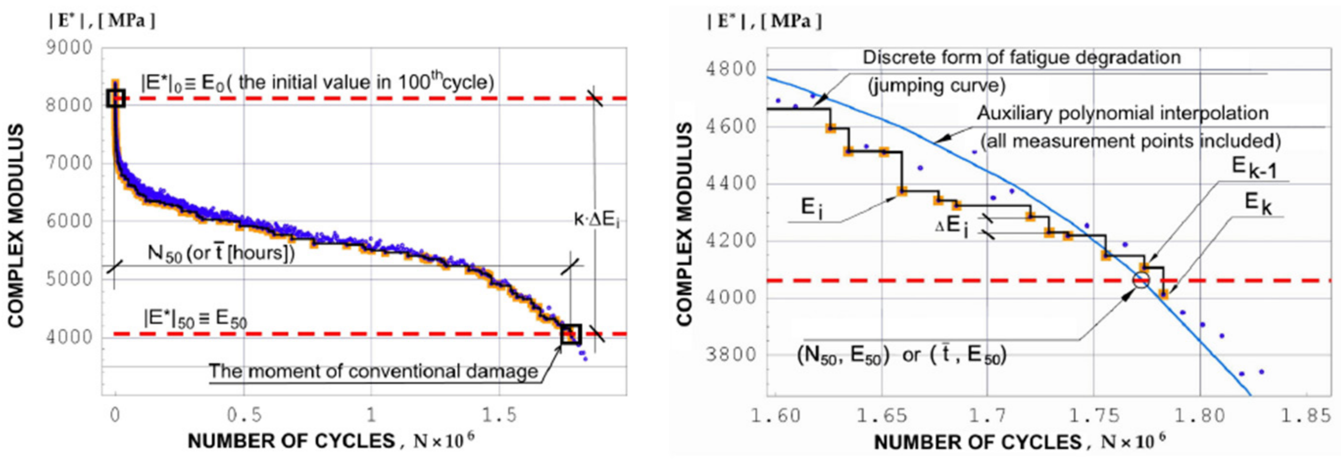

Figure 3: A detailed description of the values used for the description of fatigue degradation of HMA specimens: a) the number of elementary events (ΔEi value jumps), b) indication of coordinates in the time of conventional sample fatigue (N50, E50).

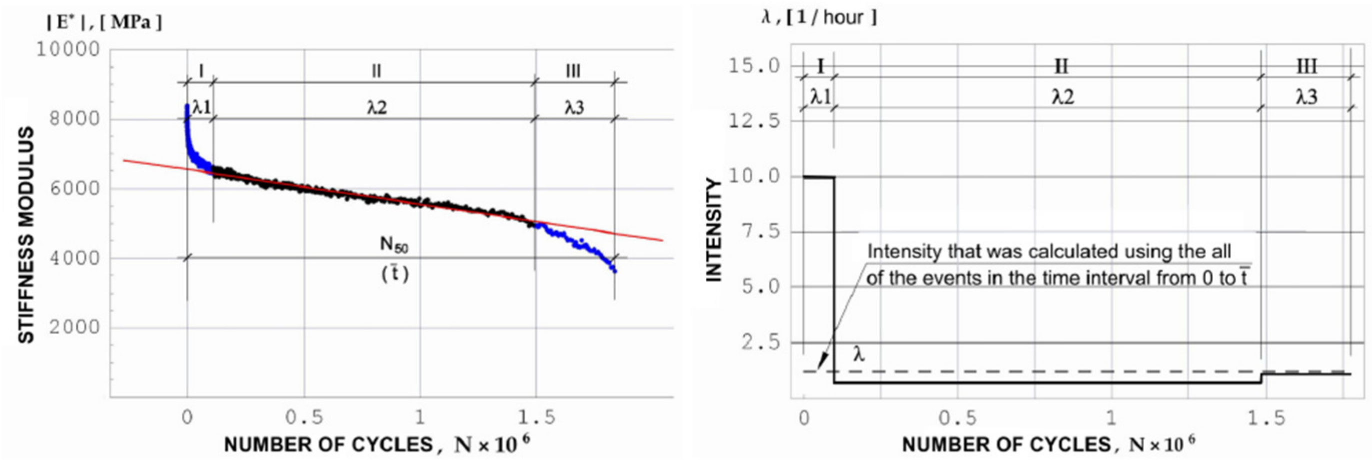

Figure 4: A comparison of λi intensity parameter: a) in individual phases of fatigue degradation curve of the complex modulus values of HMA specimens, b) changes of its values in the function of time during the test of a simple sample.
where:
k - the number of events (ΔEi jumps) within the time interval (0 ÷ t¯) λ - intensity of k events in the given time interval
t¯ - conventional critical moment in which |E∗| = 0.5 |E∗|0 = E(t¯). In the t critical moment, the number of k events has a finite character. In analyzed example (3), in the range 0 ÷ t¯, the k event occurred 120 times.

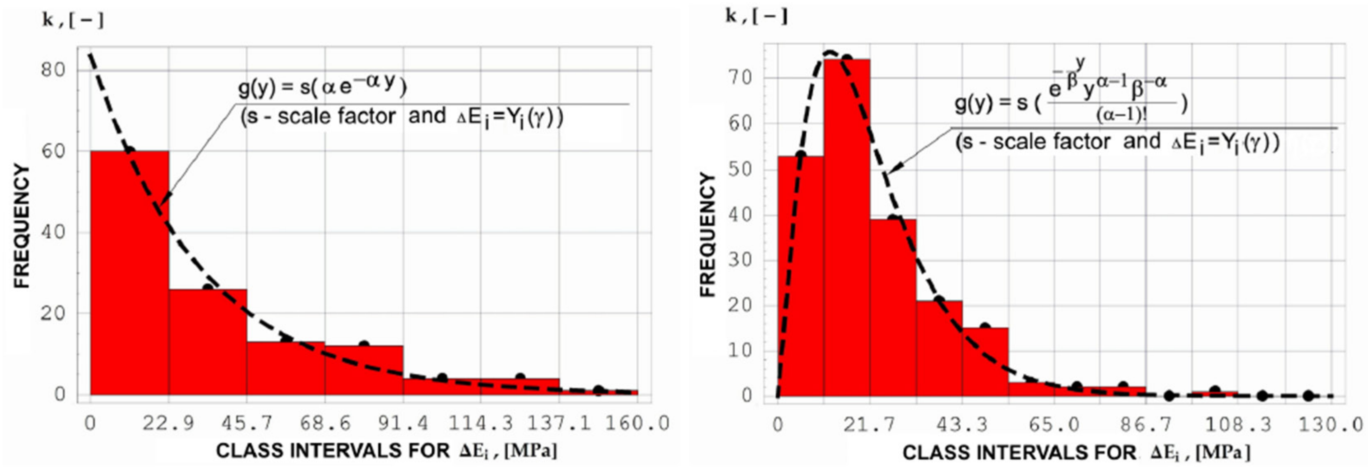

Figure 5: The typical samples of the PDFs of elementary values of ΔEi jumps, developed independently on the basis of fatigue test results on two different beam specimens cut from the same asphalt pavement base of the test section.

Results

Assuming the validity of the hypothesis that the construction of a random step curve can be treated as a discrete form of fatigue degradation of a HMA specimen, it is possible to determine the intensity parameter of the Poisson process. On the other hand, through statistical analysis of elementary value changes of the complex modulus for specific HMA specimens, one obtains the shape of the probability density functions for those values. As a result, the parameters of distributions generated in cumulative models can be determined. The sample solution generated in relation to both the modulus and the fatigue life of one HMA beam specimen is presented in Figure 6. Precisely, the show results are based on the curve marked in the Figure 6a with the red color. The elementary value changes statistics come from the exponential distribution in this case.

The comparison in Figure 6 shows that a relative error value for the mean values both in reference to the PDF of the complex modulus (Figure 6b), and the conventional fatigue life endurance (Figure 6c) is lower than 10% compared to the values resulting from the conventional fatigue life limit calculated as half of the initial value of complex modulus equal to E0/2.

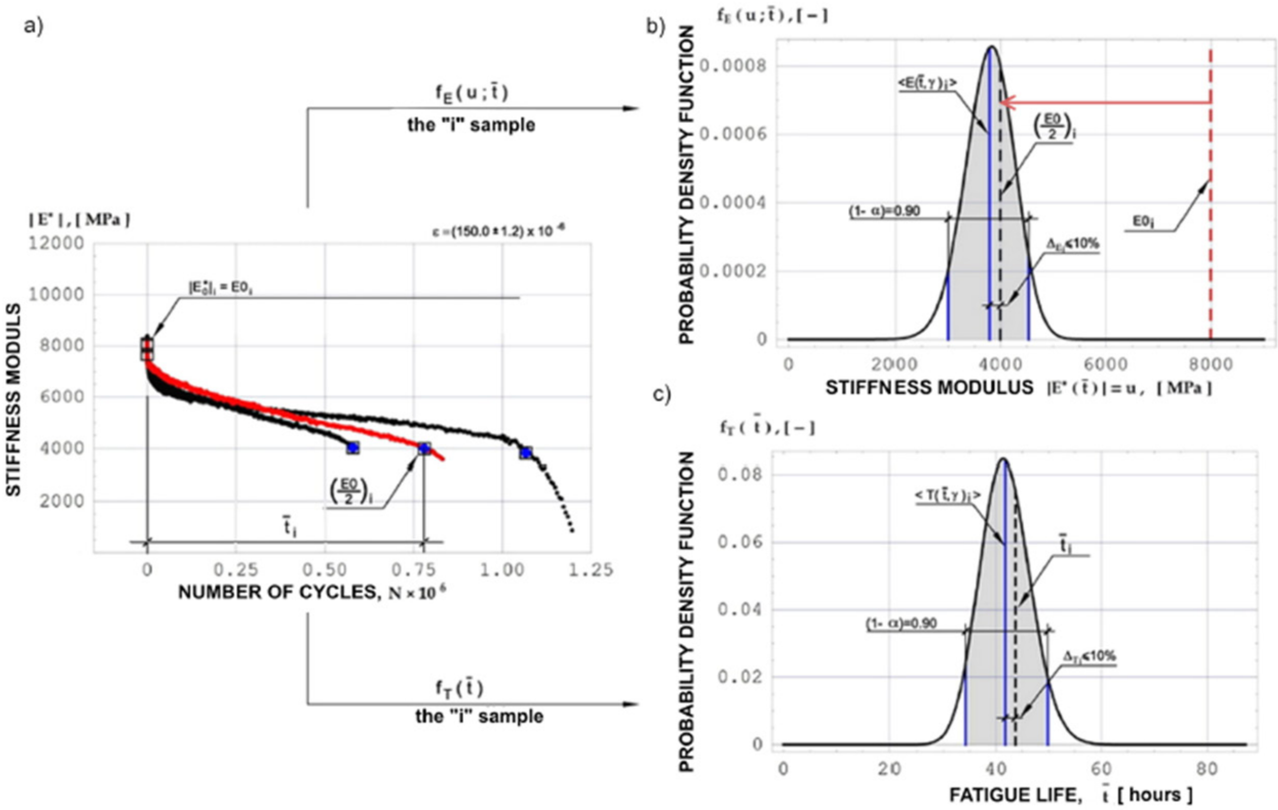

Figure 6: An example of a solution achieved for a cumulative model generated with the stationary Poisson process, a) results of the fatigue tests of three different beams, b) PDF of the complex modulus assessment, c) PDF of the HMA fatigue life estimation.

Conclusion

The experiments described in the paper show that the fatigue degradation process of the complex modulus values of the HMA specimens can be treated as a random process, which is not continuous, and has a random number of jumps of random values. Such conditions provide a possibility to use cumulative jump models in the fatigue life assessment of HMA specimens. It was assumed that in reference to the fatigue life test results on prismatic beams (4PBB) cut from the asphalt base of a 3-layer pavement of a test section, the randomness of complex modulus values degradation in cumulative jump model can be seen by both, through a probabilistic mechanism of value changes in complex modulus from i-th moment to the value in i + 1-moment with the intensity of the stationary Poisson process and through a probability distribution of HMA specimens complex modulus elementary values. It is possible to determine the parameters of an unknown distribution of elementary value changes of the HMA specimen's complex modulus observed during the fatigue degradation process by means of its statistical analysis. It was agreed that the presumable shape of the PDF for those values is described with an exponential function or a gamma function. Assuming that the intensity should be understood as a relation of the number of elementary changes (jumps) of the HMA specimen's complex modulus values to the conventional value of fatigue endurance for a given specimen, expressed in hours, it is possible to estimate the intensity parameters for a stationary Poisson process.

Cumulative jump models o er an interesting alternative to the laboratory assessment of the HMA specimens fatigue life, which combines the advantages of a classical statistical take on fatigue test results with a mathematical description of stiffness degradation of HMA (the mechanism of accumulation of fatigue cracks) as a result of cyclic loads. It is especially meaningful, as the fatigue life degradation of the HMA specimen's complex modulus values is not a stable structured process. Each degradation curve (|E(t)∗|) is characterized by short intervals of rest, and by assuming the concept of discrete description it can be concluded that between individual jumps, which gradually decrease the value of the complex modulus, nothing happens. Finally, it is important to note that the suggested approach is also worth considering in terms of the possibility to use the smaller number of specimens necessary for the laboratory tests. In the case of fatigue tests on HMA specimens, it could have a tremendous practical significance.

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Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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