

Mathematical Approach for the Prediction of Compression Pressure

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Introduction

Based on the theory of Laplace's Law that was developed to relate the wall tension and radius of cylinders (e.g. blood vessels) to the pressure difference due inflation and deflation of two halves of cylindrical vessels [1-3]. The equation can be expressed as

$$P = \frac{T}{R} \quad (1)$$

Where; P denotes pressure (Pa), T is the tension in the cylinder wall (force per unit length; newton/meter) and R is the radius of the cylinder (m).

Leung WY et al. [4] designed a new model for compression pressure based on the Laplace's law. Objective of their research was to design pressure prediction model incorporating different factors as given below

$$p = \frac{2\pi EA\epsilon}{133..2\ell_0(1+\epsilon)C} \quad (2)$$

Dubuis L et al. [5] studied patient specific FE model leg under elastic compression and design a model to evaluate compression pressure. They established a theoretical model that is given below

$$P = \frac{Stiff \epsilon}{R} \quad (3)$$

The main objective of current research includes modification of Laplace's law; development of mathematical model for prediction of compression pressure by incorporating some new parameters including true stress, true strain, true modulus, and deformed width etc.

Experimental

Materials

All of the socks samples were evaluated for their built-in physical and technical specifications as shown in Tables 1 & 2.

Table 1: Physical specifications of compression socks.

Sr.#	Brand	Sample Code	Circumference at Ankle [cm]	Fiber Analysis [%] Polyurethane/ Polyamide	Category
1	Maxis	A1	19	30/70	CCLI (2.40-2.80 kPa)
2	Aries	A2	18.6	31/69	
3	Variteks	A3	14.4	28/72	
4	Variteks	B1	15.6	33/67	CCLII (3.06-4.27 kPa)
5	Variteks	B2	17.8	30/70	
6	Variteks	B3	16.4	25/75	
7	Sigvaris	C1	16.2	50/50	CCLIII (4.53-6.13 kPa)
8	Sigvaris	C2	15.6	45/55	
9	Maxis	C3	14.6	38/62	
10	Variteks	C4	17.8	28/72	
11	Aries	C5	15.6	40/60	
12	Maxis	C6	14.6	32/68	
13	Aries	C7	16.2	45/55	

*CCL= Compression class level

Table 2: Technical specifications of compression socks.

Code	Thickness [mm]	Fabric GSM [g/m ²]	Course Density [per cm]	Wales Density [per cm]	Stitch Density [Stitches/ cm ²]	Main Yarn	Inlaid Yarn
						Type	Type
A1	0.4	139.44	22.4	19.21	430.43	MF*	DCV*
A2	0.46	134	24.6	16.2	398.52	ACV*	DCV*
A3	0.54	149.28	18.2	20	360	MF*	DCV*
B1	0.9	291.6	22	18	396	ACV*	SCV*
B2	0.75	298	22.6	18.27	412.9	MF+ACV*	DCV*
B3	0.64	306.08	23.2	22.06	511.792	MF+ACV*	DCV*
C1	0.69	281.6	20.8	22.41	466.12	MF+ACV*	SCV*
C2	0.68	265.2	21.8	20.34	443.41	MF+ACV*	SCV*
C3	0.65	296	21	23.44	492.24	MF+ACV*	DCV*
C4	0.86	360.56	19.2	19	364.8	MF+ACV*	DCV*
C5	0.7	298.44	24	22	528	MF+ACV*	DCV*
C6	0.87	312.8	16.8	24.48	411.26	MF+ACV*	DCV*
C7	0.72	384.88	22.6	26	587.6	MF+ACV*	DCV*

*MF=Multi-filament yarn, *ACV=Air covered yarn, *SCV=Single covered yarn, *DCV= Double covered yarn.

Methods

Measurement of compression pressure: Evaluation of compression pressure was done using MST MKIV (Salzmann) as shown in Figure 1.

Derivation of Mathematical Model

Theoretical background

Figure 1 is describing the mechanism of force of exertion from internal side of circular stretched cut strips per unit area of small

arc length ($dL = R.d\theta$) and reversal force of exertion assumed to be interface compression pressure (P). Due to static equilibrium condition

$$\sum \vec{F}_y = \vec{0}$$

Total sum of forces will become

$$2\vec{F} = \int_0^\pi \vec{P}_w R d\theta$$

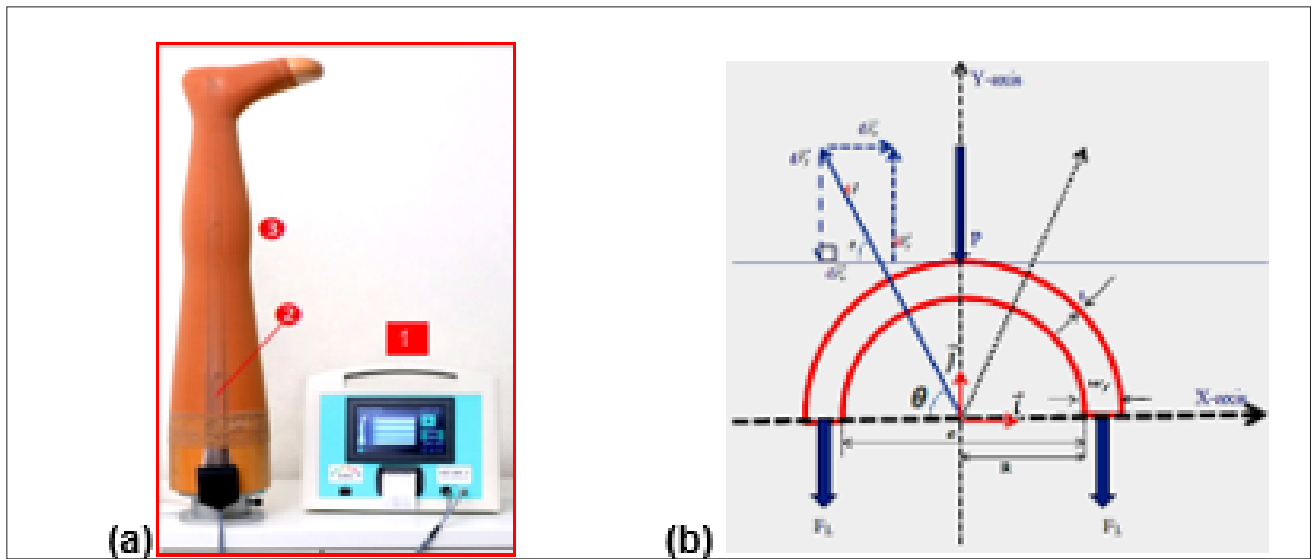


Figure 1: (a) MST Pressure measuring device (b) Mechanism of suppression of circular cut-strip due to wooden leg.

Where \bar{P} can be replaced by $P \cdot \sin \theta$ so above equation will become

$$\overline{2F_L} = PRw_f \int_0^\pi \sin \theta d\theta = P \cdot R \cdot w_f \cdot [\cos 180^\circ - \cos 0^\circ]$$

$$F_L = PRw_f$$

Here F_L is stretching force (circumferential force) of cut-strip around the leg, P is pressure [N/mm^2], R is radius of wooden leg, w_f = deformed width of socks strip around the leg [6].

Relationship between engineering Young's modulus (E.Y.M) and compression pressure (Ps) Engineering modulus

Take an ideally elastic material satisfying Hooke's law and let σ_E be the engineering stress and ϵ_E be an engineering strain Then, Young's modulus E is defined as the ratio of stress to strain so we can write.

$$E_E = \frac{\text{Engineering Stress}}{\text{Engineering Strain}} = \frac{\sigma_E}{\epsilon_E} = \frac{F_L}{A_0 \epsilon_E} \quad \text{----- (5)}$$

Here σ_l is engineering stress; ϵ_l is engineering strain while E_l is modulus of elasticity [7]. Comparing equation (4) and equation (5), we can get

$$F_L = F_L$$

$$P_E = \frac{2\pi E_E A_0 \epsilon_E}{C w_f} \quad \text{----- (6)}$$

Relationship between true Young's modulus of elasticity (T.Y.M) and compression pressure (Ps) True elastic modulus/ Young's logarithm modulus

$$E_T = \frac{\text{True Stress}}{\text{True Strain}} = \frac{\sigma_T}{\epsilon_T} = \frac{\sigma_E (1 + \epsilon_E)}{\ln(1 + \epsilon_E)} \quad \text{----- (7)}$$

Equation (4) and (7) can be modified to

$$F_L = \frac{E_T A_0 \ln(1 + \epsilon_E)}{(1 + \epsilon_E)} \quad \text{----- (8)}$$

Equating equation (1) and equation (8), relation will become

$$F_L = F_L$$

$$P_T = \frac{2\pi E_T A_0 \ln(1 + \epsilon_E)}{(1 + \epsilon_E) C w_f} \quad \text{----- (9)}$$

Equation (9) can be named as Model 1 with respect to true Young's modulus (T.Y.M)

Calculation of pressure measurement , experimental measurement of compression

In order to measure compression pressure we put on the socks samples on wooden leg keeping the MST Salzmans sleeve between wooden leg and sock samples are given in Table 3. Force Vs practical extension results were measured using Testometric tensile tester for calculation of stress and strain values.

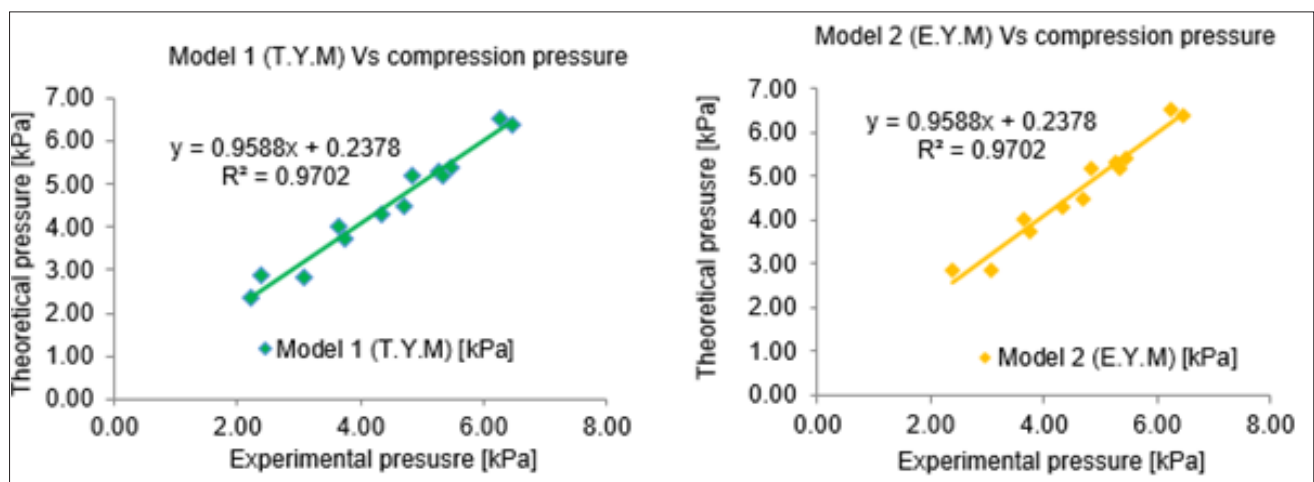
Table 3: Measurement of engineering stress, strain, modulus and theoretical compression (PE).

Sample Code	Longitudinal/ Circumferential/ Engineering Strain	De- formed Width on leg	Engineering Stress	Engineering Modulus	Model 2 (E.Y.M)	Experimen- tal Pressure	True stress [ma- chine]	True strain	True Modulus	Model 1 (T.Y.M)
	No unit	[mm]	[N/m ²]	[N/m ²]	[kPa]	[kPa]	[N/mm ²]		[N/mm ²]	[kPa]
	ϵ_e	W_f	σ_e	E_e	P_e	P_s	σ_t		E_t	P_t
A1	0.263	44.3	0.198	0.753	2.34	2.24	0.25	0.234	1.071	2.34
A2	0.29	43	0.221	0.76	2.87	2.4	0.285	0.255	1.117	2.872
A3	0.667	36	0.182	0.273	2.83	3.07	0.303	0.511	0.593	2.832
B1	0.538	48	0.151	0.28	4	3.65	0.232	0.431	0.538	3.999
B2	0.348	44	0.167	0.48	3.73	3.75	0.225	0.299	0.754	3.726
B3	0.463	48	0.228	0.492	4.29	4.34	0.334	0.381	0.876	4.294
C1	0.481	49	0.225	0.467	4.47	4.71	0.333	0.393	0.847	4.472
C2	0.538	42	0.255	0.474	5.19	4.83	0.393	0.431	0.912	5.193
C3	0.644	46.5	0.279	0.434	5.39	5.46	0.459	0.497	0.924	5.395
C4	0.348	45	0.207	0.594	5.18	5.33	0.279	0.299	0.934	5.178
C5	0.538	47.8	0.277	0.515	5.29	5.29	0.427	0.431	0.99	5.295
C6	0.644	45	0.258	0.4	6.52	6.26	0.424	0.497	0.852	6.518
C7	0.481	51.5	0.317	0.659	6.39	6.46	0.47	0.393	1.197	6.388

Results and Discussion

Measuring results of engineering stress (σ_e), true stress (σ_t), circumferential strain (ϵ_e), engineering modulus (E_e), true modulus (E_t) along with theoretical engineering pressure (P_e), experimental pressure (PS) results are given in Table 3. While deformed width (w_f) was measured results by donning the sliced cut strip in Table 3.

Figure 2 portray that the coefficient of determination value (R-square values) of Model 2 (E.Y.M) explaining 97.02% of experimental results. Figure 2 portray that the coefficient of determination value (R-square values) of Model 1 (T.Y.M) explaining 97.02% of experimental results.

**Figure 2:** Linear regression analysis of Model 1(T.Y.M) and Model 2 (E.Y.M) pressure.

Conclusion

In this research we have : Modified the Laplace law by incorporating the new term of deformed width (w_f). We developed the two new models; Model 1 (T.Y.M) as well as model 2 (E.Y.M) by incorporating some new parameters including true stress (σ_t),

true/logarithm strain (ϵ_t), true modulus of elasticity (E_t), deformed width (w_f) along with measurement of engineering stress (σ_e), engineering strain (ϵ_e) and engineering modulus of elasticity (E_e) at b position (ankel, minimum girth area) expressions into Hook's Law and modified equation of Laplace's law. Statistical validation

of newly developed models using linear analysis technique comparison to experimentally measured compression pressure results.

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