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Review Article

Innovation in the Engineering Field and Basic Sciences in the Analytical Solution of Nonlinear Partial Differential Equations by New Methods ASM and IAM

Akbari MR[1]*, Sara Akbari[2] and Esmaeil Kalantari[3]

[1]Department of Civil Engineering and Chemical Engineering, University of Germany, Germany

[2]Department of Pharmaceutical Technical Assistant, Dr. Kurt Blindow Vocational School, Germany

[3]Department of Chemical Engineering, Islamic Azad University, Iran

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***Corresponding author:** Akbari MR, Department of Civil Engineering and Chemical Engineering, University of Germany, Germany

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Abstract

In this article, we want to solve an analytical nonlinear partial differential equation that it is very applicable in all fields of engineering and basic sciences. This kind of nonlinear partial equations is used extensively in most engineering fields including chemical, mechanical, civil, electrical, and other engineering. As all experts know most of engineering actual systems behavior in practical are nonlinear process and analytical scrutiny these nonlinear problems (especially nonlinear analytical solution of partial equations) are very difficult or sometimes impossible. Our purpose is to enhance the ability of solving the mentioned nonlinear partial differential equations at engineering fields with a simple and innovative approach which entitled ''**Akbari-Sara's Method**'' or ''**ASM**'' and ''**Integral Akbari-Method**'' or ''**IAM**''.

Keywords: Akbari-Sara's Method (ASM); Integral Akbari Method (IAM); Nonlinear partial differential equation, Analytical solution, Eigenvalues

Introduction

Partial nonlinear differential equations which arise in real-world physical problems are often too complicated to be solved exactly [1,2]. And even if an exact solution is obtainable, the required calculations may be too complicated to be practical, or it might be difficult to interpret the outcome. In fact, solving nonlinear equations can guide researchers to comprehend the physical phenomena deeply and sometimes leads them to investigate some facts which are not easily understood through common observations [3-5]. On the basis of the above explanations, here we aim to discuss nonlinear partial differential and the behavior of the physical its. In this literature, we have a governing equation for studying nonlinear behavior in engineering processes and solving it analytically with a new approaches like AGM, Akbari Ganji Method [6-13]. This method is called the abbreviation ASM (Akbari Sara's Method), which can easily solve all nonlinear problems in case analytical and is very simple and flexible. Other methods compared to ASM do not have this ability to gain the solution of the presented problem in high precision and accuracy so nonlinear differential equations such as the presented problem in this case study should be solved by utilizing new approaches. In recent years, analytical methods in solving nonlinear differential equations have been presented and created by Mohammadreza Akbari, these methods are called AKLM [14] (Akbari Kalantri Leila Method in August 2020), ASM [15,16] (Akbari Sara's Method in August 2019) and AYM [17] (Akbari Yasna's Method in April 2020), MR.AM (Mohammad Reza Akbari Method in November 2020) and IAM (Integral Akbari Method in November February 2021).

Application

We consider an unsteady state of nonlinear partial differential equation with constant physical coefficients that depends on the engineering process. The constant values $\alpha, \beta, \eta, \varepsilon, \rho, ko, p, u1, u2, uo$ depend on what field of engineering we want to solve this equation. For example, these constant values in mass transfer, heat and fluid transfer, electrical circuits, as well as in beam-column equations are defined respectively in chemical engineering, mechanical engineering, civil engineering and electrical engineering, economical and all engineering fields and basics sciences.

The governing nonlinear partial differential equation of engineering fields is expressed as follows:

$$\frac{\partial}{\partial t}U = \alpha\frac{\partial}{\partial x}\left(k\frac{\partial}{\partial x}U\right) + \beta\frac{\partial}{\partial x}U + \eta U + \varepsilon U^{p};\ k = ko\cdot(1-\rho U) \qquad (1)$$

Or: $$\frac{\partial}{\partial t}U = \alpha ko(1-\rho U)\left(\frac{\partial^2}{\partial x^2}U\right) + \alpha\rho ko\left(\frac{\partial}{\partial x}U\right)^2 + \beta\left(\frac{\partial}{\partial x}U\right) + \eta U + \varepsilon U^{p}$$

Boundary and initial condition:

$$U(0,t) = u1, U(L,t) = u2; U(x,0) = uo \qquad (2)$$

i. Solving partial nonlinear differential equation by *ASM* (Akbari-Sara's Method)

We separate equation Eq. (1) from steady and unsteady state as follows:

$$U = v(x,t) + h(x) \qquad (3)$$

After separation, the result is given by the following two differential equations for $p = 2$:

$$\frac{\partial}{\partial t}v(x,t) + \alpha ko\rho\left(\frac{\partial}{\partial x}v(x,t)\right)^2 + \alpha ko\rho v(x,t)\left(\frac{\partial^2}{\partial x^2}v(x,t)\right) - \alpha ko\left(\frac{\partial^2}{\partial x^2}v(x,t)\right) - \beta\left(\frac{\partial}{\partial x}v(x,t)\right) - \eta v(x,t) - \varepsilon v(x,t)^2 = 0 \qquad (4)$$

And

$$-\alpha ko\rho\left(\frac{d}{dx}h(x)\right)^2 - \alpha ko\rho h(x)\left(\frac{d^2}{dx^2}h(x)\right) + \alpha ko\rho\left(\frac{d^2}{dx^2}h(x)\right)uo + \varepsilon h(x)^2 - 2\varepsilon h(x)uo - \alpha ko\left(\frac{d^2}{dx^2}h(x)\right) - \beta\left(\frac{d}{dx}h(x)\right) - \eta h(x) = 0 \qquad (5)$$

Using the separation equation $v(x,t) = \phi(x)\cdot z(t)$ in Eq. (4) as follows:

$$\frac{d}{dt}z(t) = \left[\frac{\alpha ko\left(\frac{d^2}{dx^2}\phi(x)\right)}{\phi(x)} + \frac{\beta\left(\frac{d}{dx}\phi(x)\right)}{\phi(x)} + \eta\right]z(t) + \left[-\frac{\alpha ko\rho\left(\frac{d}{dx}\phi(x)\right)^2}{\phi(x)} - \alpha ko\rho\left(\frac{d^2}{dx^2}\phi(x)\right) + \varepsilon\phi(x)\right]z(t)^2 \qquad (6)$$

The coefficient z(t) in the Eq. (6) can derive the following differential equation for calculating eigenvalues λ and its function $\phi(x)$.

$$\frac{\alpha ko\left(\frac{d^2}{dx^2}\phi(x)\right)}{\phi(x)} + \frac{\beta\left(\frac{d}{dx}\phi(x)\right)}{\phi(x)} + \eta = -\lambda^2 \qquad (7)$$

Solve Eq. (7) as follows:

$$\phi(x) = \frac{\left(e^{\frac{x\sqrt{-4\alpha ko\lambda^2 - 4\alpha\eta ko + \beta^2}}{\alpha ko}}\_C1 - \_C2\right)\alpha ko}{e^{\frac{1}{2}\frac{\left(\beta + \sqrt{-4\alpha ko\lambda^2 - 4\alpha\eta ko + \beta^2}\right)x}{\alpha ko}}\sqrt{-4\alpha ko\lambda^2 - 4\alpha\eta ko + \beta^2}} \qquad (8)$$

And boundary conditions:

$$B.C: \phi(0) = 0; \phi(L) = 0 \qquad (8a)$$

The constant coefficients $\_C1 - \_C2$ are obtained from Eq. (8) and given the homogeneous boundary condition with forming the coefficients matrix for the homogeneous equation as follow:

$$mat = \begin{bmatrix} -1 & 1 \\ -e^{\frac{L\sqrt{-4\alpha ko\lambda^2 - 4\alpha\eta ko + \beta^2}}{ko\alpha}} & 1 \end{bmatrix} = 0 \qquad (9)$$

The eigenvalues λ and their functions ϕ(x) are as follows:

$$\lambda = \frac{\sqrt{\alpha ko(4\pi^2\alpha^2 ko^2 m^2 - 4L^2\alpha\eta ko + L^2\beta^2}}{2\alpha koL};\ \phi(x) = \frac{1}{\xi}(e^{-\psi x} - e^{(\Delta-\psi)x}) \qquad (10)$$

The parameters of Eq. (10) are as follows:

$$\Delta = \frac{2\pi I}{\alpha ko}\sqrt{\frac{\alpha^2 ko^2 m^2}{L^2}};\psi = \frac{\pi I}{\alpha ko}\sqrt{\frac{\alpha^2 ko^2 m^2}{L^2}} + \frac{\beta}{2\alpha ko};\xi = \frac{2\pi I}{\alpha ko}\sqrt{\frac{\alpha^2 ko^2 m^2}{L^2}} \qquad (11)$$

By selecting the physical values as follows:

$$L = 1;\alpha = 0.03;\eta = 0.01;\varepsilon = 0.01;t0 = 10;uo = 0.2;u1 = 0;u2 = 0.8;m = 1;ko = 1.3;\rho = 0.03;\beta = 0.02;p = 2 \qquad (12)$$

Analytical Solution of Nonlinear Differential Equation in Steady State Eq. (5) by the boundary conditions $h(0) = u1; h(L) = u2$ is obtained by ASM method as follows:

$$h(x) = 1.100712373x^3 - 137.5817983x^2 + 947.7798882\ln(12.x + 497.)(12.x + 497.) - 81984.81066x - 2.924535250*10^6$$

Using the orthogonality of the modes of the function φ(x) in Eq. (6), we can obtain the following

$$\frac{d}{dt}z(t) = \frac{1}{\int_0^L \phi(x)^2 dx}\left\{\alpha ko\int_0^L \phi(x)\left(\frac{d^2}{dx^2}\phi(x)\right)dx + \beta\int_0^L \phi(x)\left(\frac{d}{dx}\phi(x)\right)dx - \eta\right\}z(t)$$
$$-\frac{\alpha ko\rho}{\int_0^L \phi(x)^2 dx}\left\{\int_0^L \phi(x)\left(\frac{d}{dx}\phi(x)\right)^2 dx + \frac{\alpha ko\rho}{\int_0^L \phi(x)dx}\int_0^L \phi(x)\left(\frac{d^2}{dx^2}\phi(x)\right)dx - \frac{\varepsilon}{\int_0^L \phi(x)dx}\left(\int_0^L \phi(x)^2 dx\right)\right\}$$
$$z(t)^2; IC: z(0) = \frac{\int_0^L [uo - h(x)]\cdot\phi(x)dx}{\int_0^L \phi(x)^2 dx} \qquad (13)$$

After substituted the physical values Eqs. (12) and the eigenvalue function Eqs. (10,11), and solving it by ASM method Eq.(13), the following is obtained:

$$z(t) = 0.00770588292e^{-0.3823466448t} \qquad (14)$$

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ii. Comparing the achieved solutions by numerical method and ASM (Akbari Sara's Method)

(Figure 1) And finally the solution of the nonlinear partial differential Eq. (1) is solved analytically by ASM method as follow:

$$U(x,t)=\phi(x)\cdot z(t)+h(x)$$

$$U(x,t)=\left\{-0.3183098862e^{-0.2564102564x}\sin(3.141592654x)\right\}$$
$$[0.007026327643+1.096715988e^{(-0.3823466455)t}]+1.100712373x^{3}$$
$$-137.5817983x^{2}+947.7798882\ln(12.x+497.)(12.x+497.)$$
$$-81984.81066x-2.924535250*10^{6} \quad (15)$$

The diagram of the place-time is as follows (Figure 2).

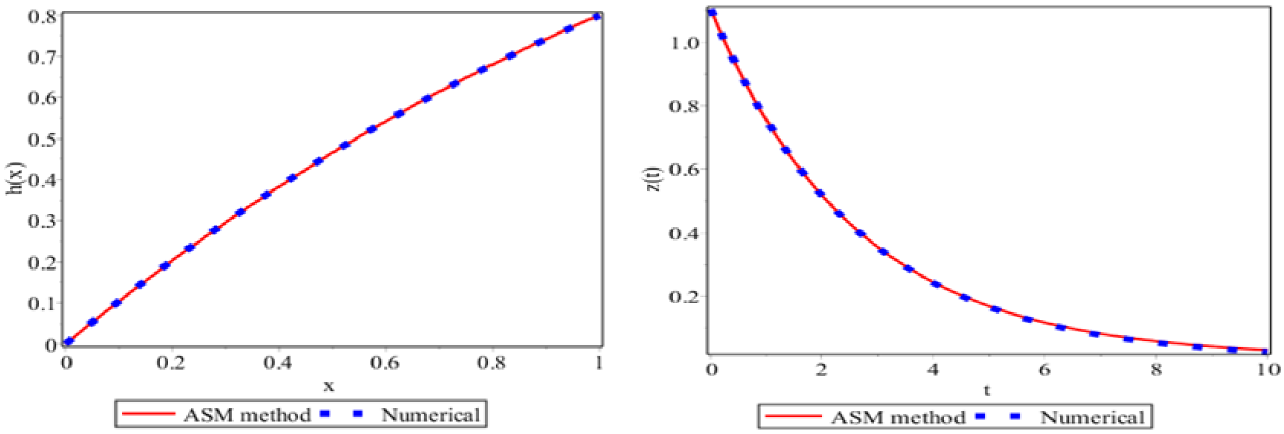

Figure 1: A comparison between *ASM* and Numerical solution.

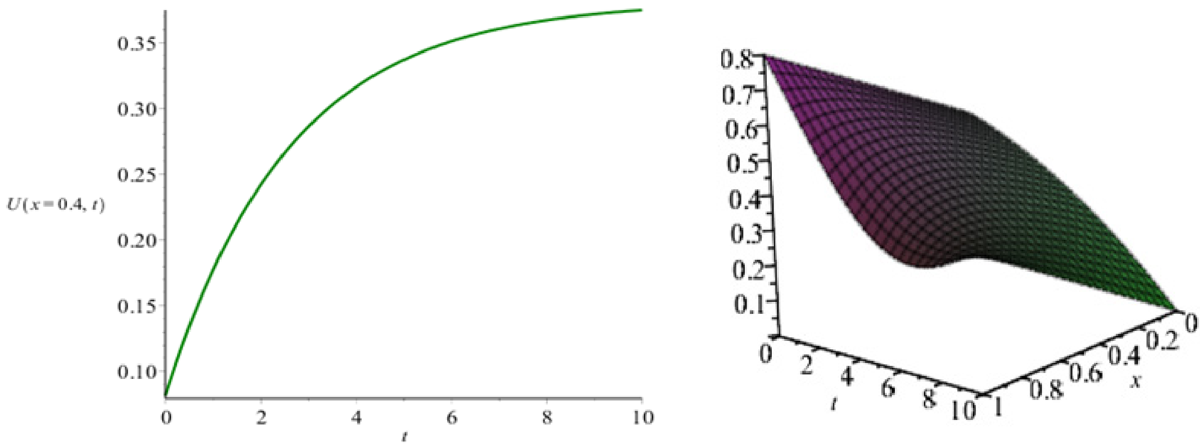

Figure 2: Figures of *ASM* solution.

iii. Solving partial nonlinear differential equation by *IAM* (Integral Akbari Method)

The governing nonlinear partial differential equation of engineering fields is expressed as follows:

$$\frac{\partial u(x,t)}{\partial t}=\alpha\frac{\partial}{\partial x}\left(k\frac{\partial u(x,t)}{\partial t}\right)+\beta\frac{\partial u(x,t)}{\partial x}+\eta u(x,t)+\varepsilon u(x,t)^{p} \quad (16)$$

$$k=ko\left\{1-\rho u(x,t)\right\} \quad (17)$$

Boundary and initial condition:

$$bc,ie:u(0,t)=u1,u(L,t)=u2,u(x,0)=uo \quad (18)$$

IAM solution process (Integral Akbari Method) The following values have been used for the physical parameters of Eqs. (12), the output answer the Eq. (16) according to boundary conditions Eqs. (17) by *IAM* method is obtained as follows:

$$u(x,t)=\frac{x}{4}\{4.419e^{-0.345436t}-1.4066+1.2108xe^{-0.345426t}$$
$$-0.37x-0.1666xe^{-0.691t}+\frac{3x^{2}}{2}[-4.54e^{-0.34544t}-1.4359$$
$$e^{-0.691t}-0.1451+0.0063-1.036308462t]\}+[-0.002355 \quad (19)$$
$$-1.036308462t+0.58011e^{-0.691t}+0.295e^{-0.3454t}+1.2982]x$$

We compare solution of *IAM* method of Eq. (19) with the numerical method (Runge-Kutte 4th), for the different times as shown below (Figure 3).

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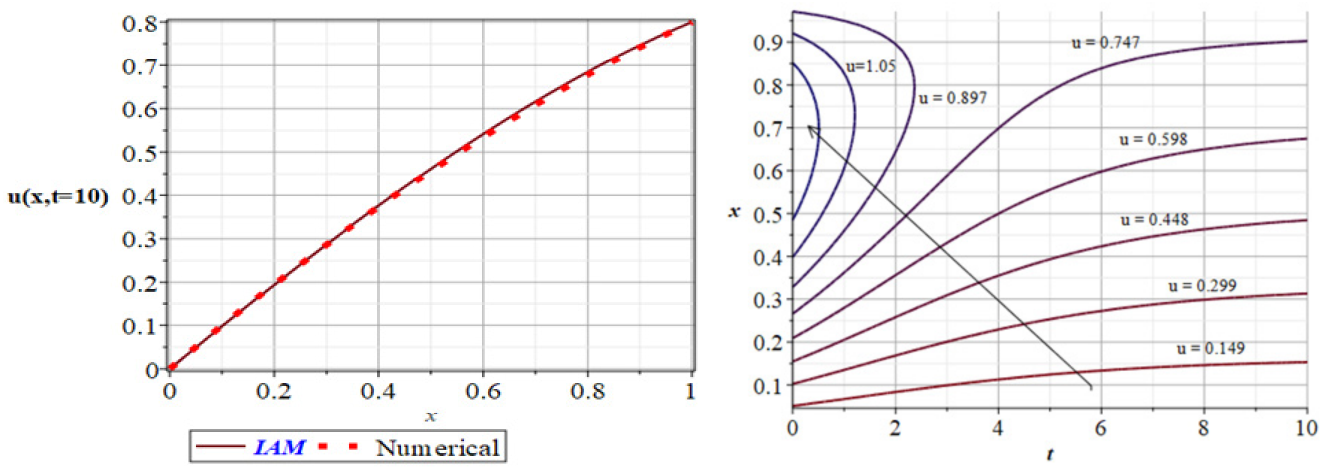

Figure 3: A comparison between *IAM* and Numerical solution and contour graphs.

Conclusion

In this paper, one complicated nonlinear partial differential equations have been introduced and analyzed completely by Akbari-Sara's Method (ASM), Integral Akbari-Method (IAM**)** and the obtained results have been compared with Numerical Method (Runge-Kutte 4th). A modern methods (ASM, IAM) for solving all kinds of complicate nonlinear partial differential equations in engineer field and basic science which can be PDEs or ODEs has been presented.

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