



Inverse Transient Thermoelastic Problems of Semi-Infinite Hollow Cylinder on Outer Curved Surface



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Submission: March 24, 2018; Published: April 02, 2018

Abstract

This paper is concerned with inverse transient thermoelastic problem of semi-infinite hollow cylinder in which we need to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem. The results are obtained in terms of Bessel's function in the form of infinite series.

Keywords: Semi-infinite hollow cylinder; Transient problem; Integral transform; Inverse problem

Introduction

In 1962, Nowacki [1] has published a book on thermoelasticity. Sierakowski & Sun [2] studied the direct problems of an exact solution to the elastic deformation of finite length hollow cylinder. In 2009, Kamdi et al. [3] studied transient thermoelastic problem for a circular solid cylinder with radiation. Walde & Khobragade [4] discussed transient thermoelastic problem of a finite length hollow cylinder. Khobragade et al. [5] derived thermal stresses of a finite length hollow cylinder. Warbhe & Khobragade [6] discussed numerical study of transient thermoelastic problem of a finite length hollow cylinder. Lamba & Khobragade [7] studied analysis of coupled thermal stresses in axisymmetric hollow cylinder. Hiranwar & Khobragade [8] studied thermoelastic problem of a cylinder with internal heat sources. Bagde & Khobragade [9] discussed heat conduction problem for a finite elliptic cylinder.

Khobragade et al. [10] investigated thermal deflection of a finite length hollow cylinder due to heat generation. Khobragade [11] studied thermoelastic analysis of a thick hollow cylinder with radiation conditions. Ghume et al. [12] derived interior thermo elastic solution of a hollow cylinder. Chauthale et al. [13] studied thermal stress analysis of a thick hollow cylinder. Singru & Khobragade [14] developed integral transform methods for inverse problem of heat conduction with known boundary of semi-infinite hollow cylinder and its stresses. Fule et al. [15] derived thermal stresses of semi-infinite hollow cylinder with internal heat source. Ghaleb et al. [16] studied a problem of plane, uncoupled linear thermoelasticity for an infinite, elliptical cylinder by a Boundary

Integral Method. Dhaba et al. [17] discussed deformation for a rectangle by a finite Fourier transform. Dhaba & Abou-Dina [18] derived thermal stresses induced by a variable heat source in a rectangle and variable pressure at its boundary by finite Fourier transform. In this paper, an attempt has been made to solve two inverse problems of thermoelasticity. In both the problems, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses on outer curved surface of semi-infinite hollow cylinder.

Statement of the Problem-I

Consider semi-infinite hollow cylinder occupying the space $D: \leq a \leq r \leq b, 0 \leq z \leq \infty$ The thermoelastic displacement function as [1] is governed by the Poisson's equation

$$\nabla^2 \phi = \left(\frac{1+\nu}{1-\nu} \right) a_1 T \tag{4.1}$$

$$\text{with } \phi=0 \text{ at } r=a \text{ and } r=b \tag{4.2}$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

ν and a_1 are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k} \frac{\partial T}{\partial t} \tag{4.3}$$

Subject to the initial conditions

$$T(r, z, 0) = 0 \tag{4.4}$$

The boundary conditions

$$T(a, z, t) = 0 \tag{4.5}$$

$$T(b, z, t) = g(z, t) \text{ (unknown)} \tag{4.6}$$

$$\left[\frac{\partial T(r, z, t)}{\partial z} \right]_{z=0} = 0 \tag{4.7}$$

$$\left[\frac{\partial T(r, z, t)}{\partial z} \right]_{z=\infty} = 0 \tag{4.8}$$

The interior condition is

$$T(\xi, z, t) = f(z, t) \tag{4.9}$$

Where k is the thermal diffusivity of the material of the cylinder

The radial and axial displacement U and W satisfying the uncoupled thermoelastic equations as [2] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1} \frac{\partial e}{\partial r} = 2 \frac{(1+\nu)}{(1-2\nu)} a_i \frac{\partial T}{\partial r} \tag{4.10}$$

$$\nabla^2 W + (1+2\nu)^{-1} \frac{\partial e}{\partial z} = 2 \frac{(1+\nu)}{(1-2\nu)} \frac{\partial T}{\partial z} \tag{4.11}$$

Where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \phi}{\partial r} \tag{4.12}$$

$$W = \frac{\partial \psi}{\partial z} \tag{4.13}$$

The stress functions are given by

$$\tau_{rz}(a, z, t) = 0 \quad \tau_{rz}(b, z, t) = 0 \quad \tau_{rz}(r, 0, t) = 0 \tag{4.14}$$

$$\sigma_r(a, z, t) = p_1, \quad \sigma_r(b, z, t) = -p_0, \quad \sigma_z(r, h, t) = 0 \tag{4.15}$$

where p1 and p0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations:

$$\sigma_r = (\lambda + 2G) \frac{\partial U}{\partial r} + \lambda \left[\frac{U}{r} + \frac{\partial W}{\partial z} \right] \tag{4.16}$$

$$\sigma_z = (\lambda + 2G) \frac{\partial W}{\partial z} + \lambda \left[\frac{\partial U}{\partial r} + \frac{U}{r} \right] \tag{4.17}$$

$$\sigma_\theta = (\lambda + 2G) \frac{U}{r} + \lambda \left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z} \right] \tag{4.18}$$

$$\tau_{rz} = G \left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} \right] \tag{4.19}$$

where $\lambda = \left(\frac{2G\nu}{1-2\nu} \right)$ is Lamé's constant, G is the shear modulus

and U and W are the displacement components. Equations (4.1) to (4.19) constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

Applying Fourier cosine transform to the equations (4.3), (4.4), (4.5), (4.6), (4.9) and using the conditions (4.7), (4.8) one obtains

$$\frac{d^2 \bar{T}}{dr^2} + \frac{1}{r} \frac{d\bar{T}}{dr} - p^2 \bar{T} = \frac{1}{k} \frac{d\bar{T}}{dt} \tag{5.1}$$

$$\bar{T}(r, n, 0) = 0 \tag{5.2}$$

$$\bar{T}(a, n, t) = 0 \tag{5.3}$$

$$\bar{T}(b, n, t) = \bar{g}(n, t) \tag{5.4}$$

$$\bar{T}(\xi, n, t) = \bar{f}(n, t) \tag{5.5}$$

where \bar{T} denotes Fourier cosine transform of T and n is Fourier cosine transform parameter,

$$p^2 = n^2 \pi^2$$

Applying Laplace transform to the equations (5.1), (5.3), (5.4), (5.5) and using the condition (5.2) one obtains

$$\frac{d^2 \bar{T}^*}{dr^2} + \frac{1}{r} \frac{d\bar{T}^*}{dr} - q^2 \bar{T}^* = 0 \tag{5.6}$$

$$\text{where } q^2 = p^2 + \frac{s}{k} \tag{5.7}$$

$$\bar{T}^*(a, n, s) = 0 \tag{5.8}$$

$$\bar{T}^*(b, n, s) = \bar{g}^*(n, s) \tag{5.9}$$

$$\bar{T}^*(\xi, n, s) = \bar{f}^*(n, s) \tag{5.10}$$

where \bar{T}^* denotes Laplace transform of \bar{T} and s is Laplace transform parameter.

The equation (5.6) is a Bessel's equation whose gives

$$\bar{T}^*(r, n, s) = A I_0(qr) + B K_0(qr) \tag{5.11}$$

Where A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.

Using (5.8) and (5.10) in (5.11) one obtains

$$A I_0(qa) + B K_0(qa) = 0 \tag{5.12}$$

$$A I_0(q\xi) + B K_0(q\xi) = \bar{f}^*(n, s) \tag{5.13}$$

Solving (5.12) and (5.13) one obtains

$$A = \frac{\bar{f}^*(n, s) K_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \quad B = -\frac{\bar{f}^*(n, s) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)}$$

Substituting the values of A and B in equation (5.11) and using condition (5.9) one obtains

$$\bar{T}^*(r, n, s) = \bar{f}^*(n, s) \left[\frac{I_0(qr) K_0(qa) - K_0(qr) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] \tag{5.14}$$

$$\bar{g}^*(n, s) = \bar{f}^*(n, s) \left[\frac{I_0(qb) K_0(qa) - K_0(qb) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] \tag{5.15}$$

Applying Inverse Laplace transform to the equation (5.14) one obtains

$$\bar{T}(r, n, t) = L^{-1} \left[\bar{f}^*(n, s) \bar{g}_1^*(s) \right] \tag{5.16}$$

$$\text{where } \bar{g}_1^*(s) = \left[\frac{I_0(qr) K_0(qa) - K_0(qr) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] \tag{5.17}$$

To Calculate the Inverse Laplace Transform of (5.17):

Applying inverse Laplace transform to the equation (5.17) one obtains

$$\bar{g}_1(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} \left[\frac{I_0(qr) K_0(qa) - K_0(qr) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] ds \tag{5.18}$$

Where c is greater than the real part of the singularities of the integrand

The integrand is a single valued function of s. The poles of the integrand are at the points

$$s = s_n = -k \left[p^2 + \lambda_m^2 \right]$$

Where λ_m are the positive roots of the transcendental equation

$$J_0(\lambda_m r)Y_0(\lambda_m a) - Y_0(\lambda_m r)J_0(\lambda_m a) = 0 \quad (5.19)$$

The zeros of $I_0(q\xi)K_0(qa) - K_0(q\xi)I_0(qa)$ are all real and simple.

The poles of the integrand (5.18) are at $s = -k[p^2 + \lambda_m^2]$ $n = 1, 2, 3, \dots$

Using the contour of Figure 1,

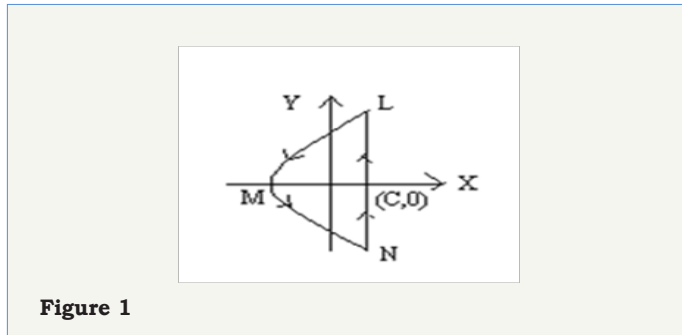


Figure 1

The integral (5.18) is equal to $2\pi i$ times the sum of the residues at the poles of the integrand.

To find the residue at the point, one requires the result:

$$\left\{ s \frac{d}{ds} (I_0(qa)K_0(q\xi) - K_0(qa)I_0(q\xi)) \right\} = \frac{a}{2kq} \left[(I_0'(qa)K_0(q\xi) - K_0'(qa)I_0(q\xi)) \right] + \frac{\xi}{2kq} \left[(I_0(qa)K_0'(q\xi) - K_0(qa)I_0'(q\xi)) \right] \quad (5.20)$$

where $s = -k[p^2 + \lambda_m^2]$, $q^2 = a_n^2 + \frac{s}{k}$ and $\frac{I_0(qa)}{I_0(q\xi)} = \frac{K_0(qa)}{K_0(q\xi)} = \frac{J_0(\lambda_m a)}{J_0(\lambda_m \xi)}$

Equation (5.20) reduces to the form

$$(p^2 + \lambda_m^2) \frac{J_0^2(\lambda_m \xi) - J_0^2(\lambda_m a)}{2\lambda_m^2 J_0(\lambda_m \xi) J_0(\lambda_m a)} \quad (5.21)$$

Also

$$\left(I_0(qa)K_0(qr) - K_0(qa)I_0(qr) \right) \Big|_{s=-k(\lambda_m^2+p^2)} = -\frac{\pi}{2} (J_0(\lambda_m a)Y_0(\lambda_m r) - Y_0(\lambda_m a)J_0(\lambda_m r)) \quad (5.22)$$

Using (5.20) and (5.22) in (5.18) one obtains

$$g_1(t) = -\pi \lambda_m^2 \frac{(J_0(\lambda_m a)Y_0(\lambda_m r) - Y_0(\lambda_m a)J_0(\lambda_m r))}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m \xi) - J_0^2(\lambda_m a))} \times J_0(\lambda_m a)J_0(\lambda_m \xi) e^{-k(\lambda_m^2+p^2)t}$$

Applying convolution theorem to the equation (5.16) one obtains

$$\bar{T}(r, n, t) = \pi \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} [J_0(\lambda_m a)Y_0(\lambda_m r) - Y_0(\lambda_m a)J_0(\lambda_m r)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' \quad (5.23)$$

Applying inverse Fourier cosine transform to the equation (5.23) we obtain

$$T(r, z, t) = \sum_{n=1}^{\infty} \cos(pz) \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' \quad (5.24)$$

Where m, n are positive integers

Applying inverse Laplace transform and then inverse Fourier cosine transform to the equation (5.15) one obtains the expression for unknown temperature gradient $g(z, t)$ as

$$g(z, t) = \sum_{n=1}^{\infty} \cos(pz) \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} \times [J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' \quad (5.25)$$

where m, n are positive integers, λ_m are the positive roots of the transcendental equation

$$J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a) = 0,$$

$$\bar{f}(n) = \int_0^{\infty} f(z) \cos(pz) dz$$

Determination of Thermoelastic Displacement

Substituting the value of temperature distribution $T(r, z, t)$ from equation (5.24) in equation (5.1), one obtains the thermoelastic displacement function $\phi(r, z, t)$ as

$$\varphi(r, z, t) = \left(\frac{1+\nu}{1-\nu} \right) \frac{\alpha}{4} \sum_{n=1}^{\infty} r^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} \times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' \quad (6.1)$$

Using the value of thermoelastic displacement function from equation (6.1) in equations (4.12) and (4.13) one obtains the radial and axial displacement U and W as

$$U = \left(\frac{1+\nu}{1-\nu} \right) \frac{\alpha}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} \times [2r(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r^2 (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' \quad (6.2)$$

$$W = -\left(\frac{1+\nu}{1-\nu} \right) \frac{\alpha}{4} \sum_{n=1}^{\infty} r^2 p \sin(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} \times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' \quad (6.3)$$

Determination of Stress Functions

Using the values of radial and axial displacement from equations (6.2) and (6.3) respectively in equations (4.16), (4.17), (4.18) and (2.19), the stress functions are obtained as

$$\sigma_r = (\lambda + 2G) \left(\frac{1+\nu}{1-\nu} \right) \frac{\alpha}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} \times [2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + 4\lambda_m r (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))] + \lambda_m^2 r^2 (J_0(\lambda_m a)Y_0''(\lambda_m r) - J_0''(\lambda_m r)Y_0(\lambda_m a))] + \lambda \left(\frac{1+\nu}{1-\nu} \right) \frac{\alpha}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))} \times [2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2+p^2)(t-t')} dt' - \lambda \left(\frac{1+\nu}{1-\nu} \right) \frac{\alpha}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (7.1)$$

$$\sigma_z = -(\lambda + 2G) \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt'$$

$$+ \lambda \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + 4\lambda_m r (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))$$

$$+ \lambda_m^2 r^2 (J_0(\lambda_m a)Y_0''(\lambda_m r) - J_0''(\lambda_m r)Y_0(\lambda_m a))] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt'$$

$$+ \lambda \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (7.2)$$

$$\sigma_\theta = (\lambda + 2G) \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt'$$

$$+ \lambda \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + 4\lambda_m r (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))$$

$$+ \lambda_m^2 r^2 (J_0(\lambda_m a)Y_0''(\lambda_m r) - J_0''(\lambda_m r)Y_0(\lambda_m a))] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt'$$

$$- \lambda \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (7.3)$$

$$\tau_{rz} = -G \left(\frac{1+\nu}{1-\nu} \right) \frac{a_t}{2} \sum_{n=1}^{\infty} p \sin(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [2r(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r^2 (J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))$$

$$\times \int_0^t \overline{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (7.4)$$

Special Case

$$\text{Set } f(z, t) = (1 - e^{-t}) e^{-z^2} (\xi - a) \quad (8.1)$$

Applying Fourier cosine transform to the equation (8.1) one obtains

$$\overline{f}(n, t) = \int_0^{\xi-a} (1 - e^{-t}) e^{-z^2} \cos(pz) dz$$

$$= \frac{(1 - e^{-t})(\xi - a)\sqrt{\pi}}{2} [e^{-p^2/4}] \quad (8.2)$$

Substituting the value of $\overline{f}(n, t)$ from equation (8.2) in the equations (5.24) and (5.25) one obtains

$$T(r, z, t) = \frac{(\xi - a)\sqrt{\pi}}{2} \sum_{n=1}^{\infty} [e^{-p^2/4}] \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t (1 - e^{-t'}) e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (8.3)$$

$$g(z, t) = \frac{(\xi - a)\sqrt{\pi}}{2} \sum_{n=1}^{\infty} [e^{-p^2/4}] \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi))}$$

$$\times [J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a)] \times \int_0^t (1 - e^{-t'}) e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (8.4)$$

Numerical Results

Set $\alpha = \frac{(\xi - a)\sqrt{\pi}}{2}$, a=1m, b=2m, $\xi=1.5m$, t=1sec, k=0.86 and λ_m are the roots of the transcendental equation

$$J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a) = 0$$

as [3] in equation (9.4) one obtains (Figure 2)

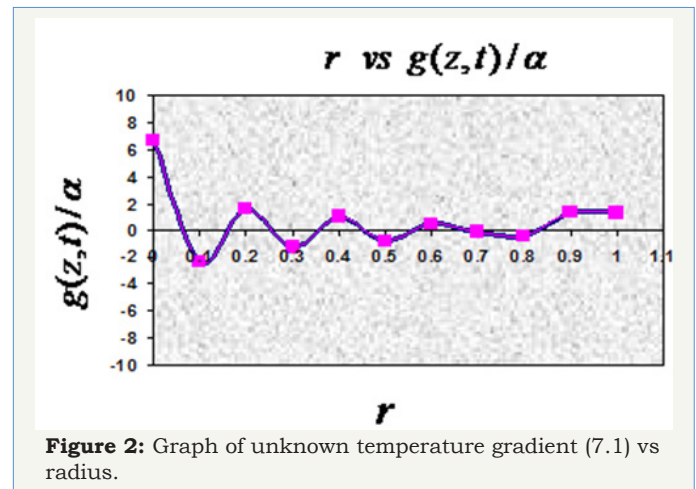


Figure 2: Graph of unknown temperature gradient (7.1) vs radius.

$$\frac{g(z, t)}{\alpha} = \sum_{n=1}^{\infty} (e^{-p^2/4}) \cos(pz)$$

$$\times \left\{ \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m) J_0(1.5\lambda_m)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m) - J_0^2(1.5\lambda_m))} [J_0(\lambda_m)Y_0(2\lambda_m) - J_0(2\lambda_m)Y_0(\lambda_m)] \right\}$$

$$\times \int_0^t (1 - e^{-t'}) e^{-0.86(\lambda_m^2 + p^2)(1-t')} dt' \quad (9.1)$$

Statement of the Problem-II

Consider semi-infinite hollow cylinder occupying the space D: $a \leq r \leq b, 0 \leq z \leq \dots$. The thermoelastic displacement function as [1] is governed by the Poisson's equation

$$\nabla^2 \phi = \left(\frac{1+\nu}{1-\nu} \right) aT \quad (10.1)$$

$$\text{with } \phi = 0 \text{ at } r=a \text{ and } r=b \quad (10.2)$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

ν and a are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k} \frac{\partial T}{\partial t} \quad (10.3)$$

Subject to the initial conditions

$$T(r, z, 0) = 0 \quad (10.4)$$

The boundary conditions are

$$T(a, z, t) = u(z, t) \tag{10.5}$$

$$T(b, z, t) = g(z, t) \text{ (unknown)} \tag{10.6}$$

$$\left[\frac{\partial T(r, z, t)}{\partial z} \right]_{z=0} = 0 \tag{10.7}$$

$$\left[\frac{\partial T(r, z, t)}{\partial z} \right]_{z=\infty} = 0 \tag{10.8}$$

The interior condition is

$$T(\xi, z, t) = f(z, t) \tag{10.9}$$

where k is the thermal diffusivity of the material of the cylinder.

The radial and axial displacement U and W satisfying the uncoupled thermoelastic equations as [2] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1} \frac{\partial e}{\partial r} = 2 \frac{(1+\nu)}{(1-2\nu)} a_t \frac{\partial T}{\partial r} \tag{10.10}$$

$$\nabla^2 W + (1+2\nu)^{-1} \frac{\partial e}{\partial z} = 2 \frac{(1+\nu)}{(1-2\nu)} \frac{\partial T}{\partial z} \tag{10.11}$$

Where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \phi}{\partial r} \tag{10.12}$$

$$W = \frac{\partial \phi}{\partial z} \tag{10.13}$$

The stress functions are given by

$$\tau_{rz}(a, z, t) = 0 \quad \tau_{rz}(b, z, t) = 0 \quad \tau_{rz}(r, 0, t) = 0 \tag{10.14}$$

$$\sigma_r(a, z, t) = p_1, \quad \sigma_r(b, z, t) = -p_0, \quad \sigma_z(r, h, t) = 0 \tag{10.15}$$

Where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations:

$$\sigma_r = (\lambda + 2G) \frac{\partial U}{\partial r} + \lambda \left[\frac{U}{r} + \frac{\partial W}{\partial z} \right] \tag{10.16}$$

$$\sigma_z = (\lambda + 2G) \frac{\partial W}{\partial z} + \lambda \left[\frac{\partial U}{\partial r} + \frac{U}{r} \right] \tag{10.17}$$

$$\sigma_\theta = (\lambda + 2G) \frac{U}{r} + \lambda \left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z} \right] \tag{10.18}$$

$$\tau_{rz} = G \left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} \right] \tag{10.19}$$

where $\lambda = \left(\frac{2G\nu}{1-2\nu} \right)$ is Lamé's constant, G is the shear modulus and U and W are the displacement components. Equations (10.1) to (10.19) constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

Applying Fourier cosine transform to the equations (10.3), (10.4), (10.5), (10.6), (10.9) and using equations (10.7), (10.8) one obtains

$$\frac{d^2 \bar{T}}{dr^2} + \frac{1}{r} \frac{d\bar{T}}{dr} - p^2 \bar{T} = \frac{1}{k} \frac{d\bar{T}}{dt} \tag{11.1}$$

$$\bar{T}(r, n, 0) = 0 \tag{11.2}$$

$$\bar{T}(a, n, t) = \bar{u}(n, t) \tag{11.3}$$

$$\bar{T}(b, n, t) = \bar{g}(n, t) \tag{11.4}$$

$$\bar{T}(\xi, n, t) = \bar{f}(n, t) \tag{11.5}$$

where \bar{T} denotes Fourier cosine transform of T and n is Fourier cosine transform parameter.

Applying Laplace transforms to the equations (11.1), (11.3), (11.4), (11.5) and using equation (11.2) we obtain

$$\frac{d^2 \bar{T}^*}{dr^2} + \frac{1}{r} \frac{d\bar{T}^*}{dr} - q^2 \bar{T}^* = 0 \tag{11.6}$$

$$\text{where } q^2 = p^2 + \frac{s}{k} \tag{11.7}$$

$$\bar{T}^*(a, n, s) = \bar{u}^*(n, s) \tag{11.8}$$

$$\bar{T}^*(b, n, s) = \bar{g}^*(n, s) \tag{11.9}$$

$$\bar{T}^*(\xi, n, s) = \bar{f}^*(n, s) \tag{11.10}$$

where \bar{T}^* denotes the Laplace transform of \bar{T} and s is a Laplace transform parameter.

The equation (11.6) is a Bessel's equation whose solution gives

$$\bar{T}^*(r, n, s) = A I_0(qr) + B K_0(qr) \tag{11.11}$$

Where A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.

Using equations (11.8) and (11.10) in equation (11.11) we get

$$A I_0(qa) + B K_0(qa) = \bar{u}^*(n, s) \tag{11.12}$$

$$A I_0(q\xi) + B K_0(q\xi) = \bar{f}^*(n, s) \tag{11.13}$$

Solving equations (11.12) and (11.13) we obtain

$$A = \frac{\bar{f}^*(n) K_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} - \frac{\bar{u}^*(n) K_0(q\xi)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)}$$

$$B = -\frac{\bar{f}^*(n, s) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} + \frac{\bar{u}^*(n, s) I_0(q\xi)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)}$$

Substituting the values of A and B in equation (11.11) and using condition (11.9) one obtains

$$\bar{T}^*(r, n, s) = \bar{f}^*(n, s) \left[\frac{I_0(qr) K_0(qa) - K_0(qr) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] \bar{T}^*(r, n, s) = \bar{f}^*(n, s) \left[\frac{I_0(qr) K_0(qa) - K_0(qr) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] \tag{11.14}$$

$$\bar{g}^*(n, s) = \bar{f}^*(n, s) \left[\frac{I_0(qb) K_0(qa) - K_0(qb) I_0(qa)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] - \bar{u}^*(n, s) \left[\frac{I_0(qb) K_0(q\xi) - K_0(qb) I_0(q\xi)}{I_0(q\xi) K_0(qa) - K_0(q\xi) I_0(qa)} \right] \tag{11.15}$$

Applying Inverse Laplace transform and inverse Fourier cosine transform to the equation (11.14) and using equation (10.6) one obtain the expressions for the temperature distribution $T(r, z, t)$

and unknown temperature gradient $g(z,t)$ as

$$T(r, z, t) = \sum_{n=1}^{\infty} \cos(pz) \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' - \sum_{n=1}^{\infty} \cos(pz) \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi)] \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (11.16)$$

$$g(z, t) = \sum_{n=1}^{\infty} \cos(pz) \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m a) Y_0(\lambda_m b) - J_0(\lambda_m b) Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' - \sum_{n=1}^{\infty} \cos(pz) \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m \xi) Y_0(\lambda_m b) - J_0(\lambda_m b) Y_0(\lambda_m \xi)] \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (11.17)$$

where m, n are positive integers, λ_m are the positive roots of the transcendental equation

$$J_0(\lambda_m a \xi) Y_0(\lambda_m b) - J_0(\lambda_m b) Y_0(\lambda_m a) = 0$$

$$J_0(\lambda_m a \xi) Y_0(\lambda_m b) - J_0(\lambda_m b) Y_0(\lambda_m a) = 0, \bar{u}(n) = \int_0^{\infty} u(z) \cos(pz) dz$$

Determination of Thermoelastic Displacement Function

Substituting the value of temperature distribution $T(r,z,t)$ from equation (11.14) in equation (10.1), one obtains the expression for thermoelastic displacement function $\phi(r, z, t)$ as

$$\varphi(r, z, t) = \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' - \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi)] \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (12.1)$$

Using the value of thermoelastic displacement function $\phi(r, z, t)$ from equation (12.1) in equations (10.12) and (10.13) one obtains the radial and axial displacement U and W as

$$U = \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2r(J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)) + \lambda_m^2 r^2 (J_0(\lambda_m a) Y_0'(\lambda_m r) - J_0'(\lambda_m r) Y_0(\lambda_m a))] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' - \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2r(J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi))] - J_0'(\lambda_m r) Y_0(\lambda_m \xi) \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (12.2)$$

$$W = -\left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p \sin(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt'$$

$$+ \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p \sin(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi)] \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (12.3)$$

Determination of Stress Functions

Using the values of radial and axial displacement U and W from equations (12.2) and (12.3) in equations (10.16), (10.17), (10.18) and (10.19), the stress functions are obtained as

$$\sigma_r = (\lambda + 2G) \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2(J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)) + 4\lambda_m r (J_0(\lambda_m a) Y_0'(\lambda_m r) - J_0'(\lambda_m r) Y_0(\lambda_m a))] + \lambda_m^2 r^2 (J_0(\lambda_m a) Y_0''(\lambda_m r) - J_0''(\lambda_m r) Y_0(\lambda_m a)) \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' + \lambda \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2(J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)) + \lambda_m r (J_0(\lambda_m a) Y_0'(\lambda_m r) - J_0'(\lambda_m r) Y_0(\lambda_m a))] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' - \lambda \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2(J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi)) + \lambda_m r (J_0(\lambda_m \xi) Y_0'(\lambda_m r) - J_0'(\lambda_m r) Y_0(\lambda_m \xi))] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' + \lambda \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi)] \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' \quad (13.1)$$

$$\sigma_z = -(\lambda + 2G) \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' + \lambda \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2(J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)) + 4\lambda_m r (J_0(\lambda_m a) Y_0'(\lambda_m r) - J_0'(\lambda_m r) Y_0(\lambda_m a))] + \lambda_m^2 r^2 (J_0(\lambda_m a) Y_0''(\lambda_m r) - J_0''(\lambda_m r) Y_0(\lambda_m a)) \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' + \lambda \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m a) J_0(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [2(J_0(\lambda_m a) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m a)) + \lambda_m r (J_0(\lambda_m a) Y_0'(\lambda_m r) - J_0'(\lambda_m r) Y_0(\lambda_m a))] \times \int_0^t \bar{f}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt' + \lambda \left(\frac{1+\nu}{1-\nu}\right) \frac{a_t}{4} \sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(\lambda_m \xi)}{(\lambda_m^2 + p^2) [J_0^2(\lambda_m a) - J_0^2(\lambda_m \xi)]} \times [J_0(\lambda_m \xi) Y_0(\lambda_m r) - J_0(\lambda_m r) Y_0(\lambda_m \xi)] \times \int_0^t \bar{u}(n, t') e^{-k(\lambda_m^2 + p^2)(t-t')} dt'$$

$$\begin{aligned}
 & -(\lambda + 2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2(J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi))+4\lambda_m r(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))] \\
 & +\lambda_m^2 r^2(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & -\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p_n^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2(J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi))+\lambda_m r(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))] \\
 & \times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \tag{13.2}
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{\theta} & = (\lambda + 2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a))] \\
 & \times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & +\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+4\lambda_m r(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a))] \\
 & +\lambda_m^2 r^2(J_0(\lambda_m a)Y_0''(\lambda_m r)-J_0''(\lambda_m r)Y_0(\lambda_m a))\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & -\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}r^2 p^2 \cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & -(\lambda + 2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2(J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi))+\lambda_m r(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))] \\
 & \times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & -\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2(J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi))+4\lambda_m r(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))] \\
 & +\lambda_m^2 r^2(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & +\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{4}\sum_{n=1}^{\infty}r^2 p^2 \cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi)]\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \tag{13.3}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{rz} & = -G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{2}\sum_{n=1}^{\infty}p \sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2r(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r^2(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a))] \\
 & \times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt'
 \end{aligned}$$

$$\begin{aligned}
 & +G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_L}{2}\sum_{n=1}^{\infty}p \sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [2r(J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi))+\lambda_m r^2(J_0(\lambda_m\xi)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m\xi))] \\
 & \times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')} dt' \tag{13.4}
 \end{aligned}$$

Special Case

$$\text{Set } f(z,t) = (1-e^{-t})e^{-z^2} \xi, u(z,t) = (1-e^{-t})e^{-z^2} a \tag{14.1}$$

Applying Fourier cosine transform to the equation (12.1) one obtains

$$\bar{f}(n,t) = \int_0^{\infty} (1-e^{-t})e^{-z^2} \xi \cos(pz) dz = \frac{(1-e^{-t})\xi\sqrt{\pi}}{2} [e^{-p^2/4}] \tag{14.2}$$

$$\bar{u}(n,t) = \int_0^{\infty} (1-e^{-t})e^{-z^2} a \cos(pz) dz = \frac{(1-e^{-t})a\sqrt{\pi}}{2} [e^{-p^2/4}] \tag{14.3}$$

Substituting the values of $\bar{f}(n,t)$ and $\bar{u}(n,t)$ from equations (14.2) and (14.3) in equations (11.24) and (11.25) one obtain

$$\begin{aligned}
 T(r,z,t) & = \frac{\xi\sqrt{\pi}}{2}\sum_{n=1}^{\infty}[e^{-p^2/4}]\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t (1-e^{-t'})e^{-k(\lambda_m^2+p^2)(t-t')} dt' \\
 & -\frac{a\sqrt{\pi}}{2}\sum_{n=1}^{\infty}[e^{-p^2/4}]\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))} \\
 & \times [J_0(\lambda_m\xi)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m\xi)]\times\int_0^t (1-e^{-t'})e^{-k(\lambda_m^2+p^2)(t-t')} dt' \tag{14.4} \\
 g(z,t) & = \frac{\xi\sqrt{\pi}}{2}\sum_{n=1}^{\infty}[e^{-p^2/4}]\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m\xi)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m\xi))}
 \end{aligned}$$

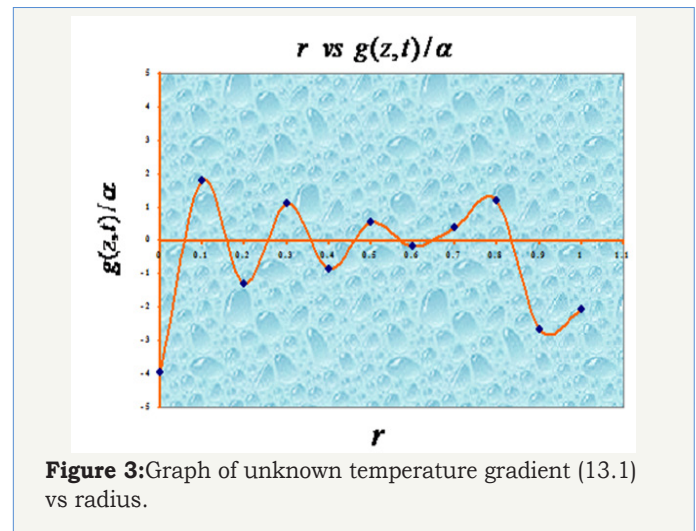


Figure 3:Graph of unknown temperature gradient (13.1) vs radius.

$$\begin{aligned}
 \frac{g(z,t)}{\alpha} & = \sum_{n=1}^{\infty}[e^{-p^2/4}]\cos(pz) \\
 & \times \left\{ \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m)J_0(1.5\lambda_m)(1.5)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m)-J_0^2(1.5\lambda_m))} [J_0(\lambda_m)Y_0(2\lambda_m)-J_0(2\lambda_m)Y_0(\lambda_m)] \right\}
 \end{aligned}$$

$$-\sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^2(1.5\lambda_m)}{(\lambda_m^2 + p^2)(J_0^2(\lambda_m) - J_0^2(1.5\lambda_m))} [J_0(1.5\lambda_m)Y_0(2\lambda_m) - J_0(2\lambda_m)Y_0(1.5\lambda_m)] \left\{ \right. \\ \left. \times \int_0^1 (1 - e^{-t'}) e^{-0.86(\lambda_m^2 + p^2)(1-t')} dt' \right. \quad (15.1)$$

Conclusion

In this section, we derived the expressions of temperature distribution, unknown temperature gradient, displacement function and thermal stresses of semi-infinite hollow cylinder on outer curved surface, when the boundary conditions are known. The Fourier cosine transform and Laplace transform techniques are used to obtain the numerical results. The solutions are obtained in terms of Bessel's function in the form of infinite series and depicted graphically. The behaviors of graphs are of sinusoidal nature. The results that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

Acknowledgement

Authors are thankful to anonymous referee for his useful suggestions.

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